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Istituto Ricerche Economico-Sociali 'Aldo Valente' - Torino

A MODEL FOR REGIONAL PLANNING APPLIED TO THE PIEDMONT REGION

by Siro Lombardini

Fourth International Conference on Input-Output Techniques
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1. Preliminary remarks (1)

A few general remarks on the use of econometric models in solving problems of economic policy may help to clarify the method and purpose of our analysis.

a) Econometric models can indicate which statistical data ought to be collected in order to produce the information we need on the economic structures and processes in which we are interested. Also the requirements which have to be satisfied in order to aggregate or compare data are thereby clarified.

b) Econometric models make it possible to compute the values of aggregated and sophisticated variables (like the total employment, the rate of growth of an industry, and so on) on the basis of values of elementary variables (labour and capital coefficients, etc.). Therefore on the basis of guesses on the elementary variables we can project future values of the other variables. This is an important result inasmuch as the values that the elementary variables are likely to assume can be visualized by technicians, while the values of the aggregated and derived variables are difficult to forecast.

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(1) The author wishes to acknowledge the contribution of T. Cozzi. The researches which have made possible the application of the model have been carried on by some research workers of IRES (Turin). In particular the researches on the ecological areas have been directed by A. Detragiache who has developed the conceptual framework as well as the methods; the researches on industrial activities by G. Bodrato; those on agriculture by G.Maspoli; the demographic analyses by M. Panero; those on public administrations by A. Prele and the researches on tourism and infrastructures by C.S. Bertuglia.

On many elementary variables various sets of guesses may look reasonable. Econometric and statistical analyses are not always adequate to produce projections of the values of some coefficients and exogeneous variables. That is the case when such values depend on political decisions or are the result of technical innovations. We are then obliged to consider various sets of reasonable guesses. The econometric models will clarify the economic and demographic implications of each of these sets.

c) When the objectives of economic policy can be expressed by the values of some magnitude that has to be reached or to be maximized, the econometric model can be transformed into a mathematical programming model (decision model). Then, according to the structure of the model, we must find out the values of some unknowns or the shape of some relations which maximise some function or functional.

Input-output models are particularly useful for the purposes pointed out under b) and c). To understand how these models can be employed to help to solve the problems of regional planning, we shall examine the goals of the plan.

2. The goals of a regional plan

The goals of a regional plan can be divided into two groups:

a) goals which can be established by solving the problems peculiar to specific sectors (exogeneous goals). Partial analyses are then sufficient to evaluate the extent to which these goals can be achieved and to determine the processes-by which such goals can be implemented-contributing to determine the values of some parameters and exogeneous variables.

b) goals that can be assessed only by analysing the direct and indirect effects of certain actions on the whole economy (endogeneous goals). To determine the optimum level of these goals, we need to develop an econometric analysis of the structure of the economy. We can confine ourselves in setting up a descriptive model to ascertain the results of alternative actions or we can express the purpose of the action by a welfare function or functional to be maximized. In the former case after our analysis, we shall adjust our action in order to get results considered satisfying in relation to the goals which we want to achieve; in the latter case a normative model has to be established to determine the optimum course of our action.

3. Exogeneous goals of the regional planning for Piedmont

In a developed region like Piedmont we can classify as goals of the first kind:

1) the achievement of a satisfactory level for the productivity of agriculture and the realization of new social and housing conditions for the population engaged in agricultural activities. Given the technological possibilities, the socio-institutional obstacles, the instruments of economic policy that can be made available, we can estimate by what changes such productivity targets can be reached.

2) Goals for the technological development of some industries which can be proposed on the basis of the actual trends and of the requirements of the international competition. In the actual stage of economic planning, the programmes of the big firms are to be considered as partial goals in framing the perspectives of separate industries in a re-

gional plan. New industrial activities can be conceived in relation to specific opportunities and requirements. Particular goals can be visualized in the industries which will be affected. The need for a more spatially balanced industrial structure may help to choose among such alternatives. The specific goals to be proposed for some industries are then linked with the general goal of the optimum physical plan of the region.. Such a plan can be framed in all the required details only when the levels of all the goals of both kinds are assessed.

Yet certain features of the plan can be proposed after some partial analyses. Those are the features suggested by the existence of productive facilities which are not conveniently exploited and by the minimum concentration required in certain strategic areas for a more balanced spatial distribution of economic activities.

Having solved the specific problems which we have mentioned we will be able to make reasonable guesses on the values of some parameters (as f.i. the coefficients of the labour productivities) and exogeneous variables (as f.i. the production of some industries which will be sold outside the region).

4. Endogeneous goals

The endogeneous goals are mainly those concerning the infrastructures. Social capital targets (hospitals,schools, roads, etc.) cannot be determined exactly on the basis of partial analyses: they depend on the various levels that economic activities can reach in the various areas which are characterized by different needs of adjustments in the social capital and by different marginal inputs of social capital for each additional inhabitant.

On the other side, the level of the various activities and therefore the degree of industrial development of the various areas cannot be assessed without taking into account the level of the demand for the commodities required to construct the infrastructures.

5. The two stages of the research for a regional planning. The sectoral analyses

To make the regional plan projections we need therefore to carry out two kinds of researches:

a) sectoral researches to single out the exogeneous goals and to produce the estimates of the parameters and coefficients characterizing the structure of the economy;

b) an econometric analysis by a particular kind of input-output model capable of dealing both with the interindustry relations and with the circular relations mentioned above, which appear as soon as we consider the spatial distribution of economic activities.

Analyses of the economic activities have thus been made mainly in order

1) to make reasonable guesses on the national and international demand for the commodities produced by the autonomous industries: i.e. industries having a national-wide market, that are present in the region because of its localization advantages, or because of historical reasons. The production sold outside the region can be evaluated by subtracting from the total production the quantity consumed inside the region, which is estimated ex ante. By an iterative process we can assure that the ex ante evaluation will correspond to the evaluation implied by the solution of the model.

2) to single out the specific problems of the various sectors and the goals of the first kind. In connection with such problems, we can determine the most efficient tools to implement the regional policy. Then, as we have seen, it is possible to establish the level at which the goals can be reached and the consequences that the economic policy may have on the coefficients and exogeneous variables.

3) to evaluate the coefficients and the parameters by which we can describe the actual structure, and to get informations on the possible changes that such parameters will undergo in the future. Such changes may be caused by exogeneous events (technical progress, for instance, which exerts a great influence on the labour input coefficients).

6. The distribution of economic activities in the ecological areas and the infrastructures

The distribution of economic activities and of population within the region has to be studied in order:

1) to determine a partition of the region in ecological areas. An ecological area is a connected zone equipped with social services and large enough to offer the facilities of the urban life to its inhabitants. Such a partition is made on the basis of the actual trends, of some technological and sociological requirements and of the effects of the policy that can be pursued by the plan to reduce the day-migrations of the workers;

2) to determine the most efficient structure of the roads and public transport facilities, linking up the region with other regions and the urban and industrial poles of the various ecological areas with one other. For Piedmont, the research has led to one system only of efficient roads and

public transportations, inasmuch as the relative importance of the various poles could be easily assessed in advance and the need for connections with other regions could be determined independently of the level of activity of the various industries of the various areas. In general the partition of the region in ecological areas and the determination of the best road and transportation system are two problems connected to each other that can be solved only by iterative processes to be carried on together with the process of solution of the model.

The partial analyses that lead to the partition of the region in ecological areas enable us to formulate a first set of estimates of the following coefficients:

1) quota of employment of the i.th industry, which is occupied in the h.th ecological area (ξ_{ih});

2) quota of active population employed in the k.th area, which resides in the h.th area (ν_{hk}).

Of course the values of these parameters can be reviewed, after the econometric model has been solved, and quantitative elements have been obtained, enabling us to decide whether the ecological structure is adequate to the productive and demographic requirements.

The analysis of the localization of the economic activities throughout the region is extremely important when we deal with a regional planning model. As a matter of fact, this analyses enables us to determine the amount of infrastructures and housing necessary in each ecological area as a consequence of the development which will occur in each one of them.

The quantities of commodities of the different industries required for the construction of the infrastructures are conspicuous especially in a region like Piedmont, where, because of the relevant migration inside the region and from other regions, the urban development is of a remarkable size.

The level of infrastructures and housing is of a paramount importance in establishing the level of expenditure of the local public administrations. One of the reasons why physical planning is needed, is the importance of external economies which are to be taken into account in evaluating the most efficient spatial distribution of economic activities and residences.

In particular the choice of the industrial plants localization should not result only from private decisions, but also from the public actions to implement the physical plan.

In the local economic policy, the problem of housing becomes more and more important because of the increasing percentage of subsidized houses construction. For this reason we deem it convenient to keep separated the demand for construction arising from housing and infrastructures needs.

Owing to the reduction of employment in agriculture, which will continue during the next years, it seems reasonable that the farmer's houses actually existing and in good conditions will be sufficient also in the future.

The housing needs, in each region, will be determined essentially by the non agricultural population increase and the quota of the present non agricultural population, living in houses which will be rebuilt, because of their sanitary and habitability conditions. If we call P_h° the non agricultural population living in good houses (i.e. houses which will not need to be rebuilt within the next 5 years) in the h.th area, and Z_h the non agricultural population at the end of the planning period, then the housing needs in the h.th area will be proportional to: $Z_h - P_h^\circ$

Supposing a standard density of one room per capita, the number of rooms which will have to be built is: $Z_h - P_h^\circ$ We shall indicate by δ_c the value of "constructions" required to build one room.

The infrastructures needed in the region are proportional to the increase of the total population. Their construction involves an activation of the building sector and we shall indicate by ω_h the value of "construction" required for the infrastructures needed for an additional unit of population in the h.th ecological area.

While it seems reasonable to think that the value of output of "construction" for one room is the same in each area (the different values of the houses depending essentially upon the different values of the sites) the amount of infrastructures needed for one additional unit of population varies according to the area.

7. The model: Coefficients and variables

The input output coefficients are defined in the usual way. Their estimate has been based a) on the actual data obtained by partial analysės through a sample for the small and medium firms and a census of the large ones and b) on the evaluation of the effects of the spontaneous developments and of the actions aiming at the achievement of the exogeneous goals.

The capital coefficients have been estimated on the basis of:

a) the data on investments and productions which have been collected for a five yearsperiod by the partial analysès;

b) the assessment of the requirements of the industries to make their structures up to date with technical progress.

The consumption coefficients express the quota of the household income which is spent in each consumption good.

All variables are expressed in 1963 monetay units. Only the level of activity of the tertiary sectors has been expressed by the number of workers employed.

Such a level depends:

a) on the number of workers employed in industries;

b) on the household income.

The coefficients of the tertiary employment for a unit of employment in industries and for a unit of household income are estimated on the basis of the actual trends and of the effects of the policy envisaged by the plan.

The commercial activities enter into the model only as a sector producing services: the commodities are supposed to come directly from the industries at a value not including the value added by trade.

As for the commercial relations of the region with the outside world, it is possible to determine only the balance between imports and exports out of the region (1). But we cannot determine the balance for the commercial sector since we cannot ascertain the values of trades of commodities acquired outside the region and to be sold in other regions. This "autonomous" activity of the commercial sector is not very important in Piedmont.

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(1) IRES - Italconsult - Sema: "Strutture e Prospettive di una Regione" - Milano, 1962, pp. 698-700.

The levels of activity of agriculture and tourism as well as their demands for goods have been estimated as exogeneous variables.

We have distinguished the propulsive industries (mostly automobile and office supplies industries) from the other mechanical industries. For the propulsive industrie we have evaluated the specific perspectives of the single firms.

The exogeneous components of the demand for the products of each industry are:

1) the demand for commodities used as capital goods by agri culture
2) the demand for intermediate products by agriculture
3) the demand for the products of construction required to make social capital up to date
4) the demand by the public Administrations
5) the demand from outside the region for the autonomous sectors
6) the demand by tourism.

Households incomes are made up by:

1) the wage bill paid by the industries and the tertiary sectors, depending on the level of their activities
2) the profits distributed by industries, which are supposed to be proportionate to their activities
3) mixed incomes earned by small entrepreneurs and craftsmen, which include both capital and labour incomes
4) predetermined incomes (not depending on the solution of the model) i.e. agricultural incomes, salaries of civil servants, incomes earned by people working outside the region (that have to be guessed before the solution of the model and then eventually adjusted through an iterative process), pensions and social transferts by the Public Administration to households.

The endogeneous variables of the model are:

1) production levels of the various industries
2) employment in tertiary sectors
3) non agricultural population of the various zones
4) household income
5) percentages of the demands for the various capital goods (houses and social capital being included) maturing over the period, that will be satisfied by the production of the last year. The demand for capital goods by tertiary sectors is supposed to be proportional to the increase in the level of its employment. Such percentages depend on:
6) the rate of growth of industrial production, of employment in the tertiary sector, of the non agricultural population living in houses not to be rebuilt and of the population of each zone (see section 9).

8. <u>The list of the symbols</u>.

A. UNKNOWNS

The unknowns expressed in monetary values are all in 1963 prices.

x_j = value of the production of the j^{th} industry in the terminal year (j = 1,2 ... 16)

y_s = number of workers in the tertiary sectors (the transport sector being excluded) in the terminal year

x_r = personal income in the terminal year

z_h = inhabitants of the h.th ecological area in the terminal year (h = 1,2 ... 15)

ε_j = quota of the investiments of the j^{th} industry for the planning period to be imputed to the terminal year (which depends on the rate of growth r_j) (j = 1,2...16)

ε_s = quota of the investments of the tertiary sectors for the planning period to be imputed to the terminal year (which depends on the rate of growth r_s of employment in these sectors)

ε_c = quota of the value of the houses to be built in the period for the non agricultural population to be imputed to the terminal year (which depends on the rate of growth r_c)

ε_h = quota of the value of the social capital (roads, hospitals, schools, etc.) required for an additional inhabitant in the h.th ecological area to be imputed to the terminal year (the quota depends on the rate of growth r_h; h = 1.2 ... 15)

r_j = yearly rate of growth of production in the j.th industry (j = 1,2 ... 16)

r_s = yearly rate of growth of employment in the tertiary sectors

r_c = yearly rate of growth of the non agricultural population living in houses not to be rebuilt in the period

r_h = yearly rate of growth of the population in the h.th ecological area (h = 1,2 ... 15)

B. PREDETERMINATED AND EXOGENEOUS VARIABLES

I) <u>Values of production, of employment and of population parameters at the initial year</u>

1) x_j^o = current value of production in industry j (j = 1,2 ... 16)

2) y_s^o = employment in tertiary sectors

3) o_{ah}^o = number of workers employed in the agriculture in the h.th ecological area

4) o_d° = employment in agriculture

5) p_h° = non agricultural population of the h.th ecological area living in houses which do not need to be built or rebuilt in the period

6) z_h° = non agricultural population living in the h.th ecological area

7) σ_h° = ratio between total population and active population in the h.th ecological area.

II) Employment at the terminal year in the exogeneous sectors

1) o_{ah} = number of workers employed in the agriculture of h.th ecological area

2) o_a = ($= \sum_h o_{ah}$) total employment in agriculture

3) o_t = number of workers employed in the tourism sector

4) o_e = number of workers living in the region and working outside the region

5) o_d = number of workers and employees of the Public Administrations

6) ζ_{dh} = quota of the workers and employees of the Public Administrations living in the h.th ecological area.

III) Incomes at the terminal year per unit of employment in the exogeneous sectors

1) P_{na} = net product per worker in agriculture

2) s_d = wage rate in the Public Administration sector

3) s_e = wage rate per worker employed outside the region.

IV) Other exogeneous incomes at the terminal year

1) T_h = social transferts from the "Comuni" Administration of the h.th ecological area to the households

2) T_d = social transferts from the "Provincie" Administrations to the households

3) P = pensions paid to the households.

V) Exogeneous components of the final demand

1) D_{i1} = current expenditure on good i by the tourism sector

2) D_{i2} = current expenditure on good i for autonomous industrial investments

3) D_{i3} = current expenditure on good i for investment by agriculture

4) D_{i4} = current expenditure on single use good i by agriculture

5) D_{i5} = current expenditure on good i for investment in social capital adjustment

6) D_{i6} = current expenditure on good i by the Public Administration sector

7) D_{i7} = exports of good i.

VI) Number of years in the period

1) T = number of years in the period.

C. COEFFICIENTS

I) Technical coefficients for the intermediate commodities and for labour, and consumption coefficients (values at the terminal year)

1) a_{ij} = monetary value of commodity i used for the production of a unit value of the commodity by industry j (i, j = 1,2 ... 16)

2) a_{is} = monetary value of commodity i used by the tertiary sectors for the employment of one worker (i = 1,2 ... 16)

3) o_j = quantity of labour employed for the production of a unit value of commodity by industry j (j = 1,2 ... 16)

4) o_s = quantity of labour employed in the tertiary sectors which are complementary to the industry for the employment of one industrial worker

5) o_{sr} = quantity of labour employed in the commercial sectors for the activity required in connection with the expenditure of a unit value of the personal income

6) c_i = quota of the personal income spent on commodity i (i = 1,2 ... 16).

II) <u>Technical coefficients for capital goods, houses and social capital (values at the terminal year)</u>

1) b_{ij} = monetary value of commodity i employed as a capital good for the production of one unit of commodity j (i,j = 1,2 ... 16)

2) b_{is} = monetary value of commodity i employed as a capital good in the tertiary sectors in connection with the employment of one worker (i = 1,2 ... 16)

3) ω_h = value of the social capital (hospitals, schools, roads, etc.) which is required for an additional inhabitant in the h.th ecological area (h = 1,2 ... 15)

4) δ_c = value of the houses required for one additional unit of the non agricultural population.

III) <u>Commercial coefficients</u>

1) α_{ij} = quota of the input a_{ij} supplied by firms of the region (i,j = 1,2 ... 16)

2) α_{is} = quota of the input a_{is} supplied by firms of the region (i, = 1,2 ... 16)

3) β_{ij} = quota of the input b_{ij} supplied by firms of the region (i,j = 1,2 ... 16)

4) $\hat{\beta}_{is}$ – quota of the input b_{is} supplied by firms of the region (i = 1,2 ... 16)

5) γ_i – quota of the consumption of commodity i for one unit of personal income, which is supplied by firms of the region (i = 1,2 ... 16)

6) e_i – quota of the production of sector i which is exported outside the region.

IV) Income coefficients (values at the terminal year)

1) s_i = wage rate of industry i (i = 1,2 ... 16)

2) s_s = wage rate in the tertiary sectors

3) π_i = distributed profit margin of industry i (i = 1,2 ... 16)

4) π_s = distributed profit per worker in the tertiary sectors

5) μ_i = distributed mixed (capital and labour) income per unit of production in industry i (i = 1,2 ... 16)

6) μ_s = distributed mixed (capital and labour) income per worker in the tertiary sectors.

V) Coefficients for the allocation of economic activities among the ecological areas

1) ξ_{ih} = quota of the production of industry i localized in area h (i = 1,2 ... 16, h = 1,2 ... 15)

2) ξ_{sh} = quota of the tertiary activities localized in area h (h = 1,2 ... 15)

3) ν_{hk} = quota of the population increase induced by the industrial development of area k, which resides in area h (h,k = 1,2 ... 15).

VI) Population coefficients

1) σ_h = ratio between total population and active population in area h (h = 1,2 ... 15)

2) ϱ_j = ratio between active industrial population of the sector j (employed or unemployed) and the number of workers employed in industry j (j = 1,2...16)

3) ϱ_s = ratio between active population of the tertiary sectors and the number of workers employed in the same sectors

4) ϱ_d = ratio between active population in the Public Administration and the number of workers employed in the same sector.

VALUES OF THE EXOGENEOUS VARIABLES AND COEFFICIENTS

B - PREDETERMINED AND EXOGENEOUS VARIABLES

I. 2 : y_e^o = 309.521;
I. 4 : o_d^o = 82.663;
II. 2 : o_a = 275.000;
II. 3 : o_t = 22.974;
II. 4 : o_e = 13.000;
II. 5 : o_d = 99.164;
III. 1 : p_{na} = 0,01124000;
III. 2 : s_d = 0,01836265;
III. 3 : s_e = 0,01500000;
IV. 3 : P = 3555,18;
VI. 1 : T = 7;
IV. 2 : T_d = 11,12.

B — PREDETERMINED AND EXOGENEOUS VARIABLES

Industries	I. 1 x^*_j	V 1 D_{i1}	V 2 D_{i2}	V 3 D_{i3}	V. 4 D_{i4}	V 5 D_{i5}	V 6 D_{i6}	V. 7 D_{i7}
1. Propulsive ind.	10.624,70			22,47				17.496 73
2. Metals engineering	10 015,21			15 00	70,00		502,88	9 092,21
3. Textiles	4.297,10						54,65	4.178,36
4. Clothing	1.229,90	22,00					59 62	920,92
5. Chemicals	2.880 30				40 00		425,53	2.712,13
6. Food and drink	3.405,80	43,00			104 00		178,59	2 884,36
7. Leather	419,40							420,30
8. Pulp mills and paper	687,80						31,92	428,22
9. Non metallic mining and mineral manufactures	1.417 60						121,56	500 00
10. Timber Furniture	969,60				13,00		23,80	424 26
11. Rubber, tires and cables	1.517,30						17,26	117 52
12. Polygraphic Ind. and Publishing	720,00	11,00					53,00	100 00
13. Other manufactures	471,40	11,00						109,00
14. Construction	4.197,60	336,19		160 00	30,00	1.424 50	437,24	
15. Electricity, gas and water	1.333,90				50,00		23,64	
16. Transport and communications	2.496,00	44,00					97,44	203,00(1)

(1) - Public financial contributions

B — PREDETERMINED AND EXOGENEOUS VARIABLES

Zones	I. 3 o^{*}_{ah}	I. 5 p^{*}_{h}	I. 6 z^{*}_{h}	I. 7 σ^{*}_{h}	II. 1 o_{ah}	II. 6 ξ_{dh}	IV. 1 T_h
I - Torino	58.500	1.415.678	1.677.344	2,22	47.000	0,5181	3,01
II - Ivrea	12.900	61.916	81.148	2,11	10.000	0,0192	0,46
III - Pinerolo	13.900	67.771	83.155	2,19	11.000	0,0183	0,45
IV - Vercelli	19.800	61.135	79.915	2,10	16.000	0,0392	0,38
V - Borgosesia	5.000	53.631	71.129	2,06	4.000	0,0105	0,24
VI - Biella	6.900	134.348	169.846	1,99	6.000	0,0235	0,51
VII - Novara	20,300	164.689	213.882	2,18	15.000	0,0687	0,70
VIII - Verbania	7.500	156.997	185.576	2,27	6.000	0,0379	0,81
IX - Cuneo	24.800	64.134	84.276	2,25	20.000	0,0451	0,38
X - Saluzzo - Savigliano - Fossano	27.800	51.767	73.953	2,30	23.000	0,0234	2,05
XI - Alba - Bra	25.800	58.947	70.426	2,26	19.000	0,0231	0,61
XII - Mondovì	19.800	40.336	52.087	2,28	15.000	0,0203	0,42
XIII - Asti	39.700	91.989	110,299	2,18	32.000	0,0394	0,78
XIV - Alessandria	50.100	232.985	283.092	2,33	36.000	0,0943	6,82
XV - Casale Monferrato	19.300	53.037	64.917	2,21	15.000	0,0190	0,31

C - COEFFICIENTS

I - LABOUR COEFFICIENTS (FOR THE TERTIARY SECTORS)

4) $o_s = 0{,}092871;$

5) $o_{sr} = 6{,}8948.$

II - HOUSE CONSTRUCTION INPUT

4) $\delta_e = 0{,}01236750.$

IV - INCOME COEFFICIENTS

2) $s_s = 0{,}00534960;$

4) $\pi_e = 0{,}01026703;$

6) $\mu_e = 0{,}01247897.$

VI - POPULATION COEFFICIENTS

3) $'\rho_e = 1{,}050;$

4) $\rho_d = 1{,}000.$

C - COEFFICIENTS

Zones	CII - Social capital coefficients 3) ω_h	CV - Coeff. for alloc. of tertiary employ. among zones 2) ξ_{sh}	CVI - Population coefficients 1) σ_h
I - Torino	0,0035152	0,5241	2,25
II - Ivrea	0,0030497	0,0244	2,17
III - Pinerolo	0,0030936	0,0225	2,25
IV - Vercelli	0,0029200	0,0293	2,24
V - Borgosesia	0,0031115	0,0147	2,08
VI - Biella	0,0030470	0,0398	2,05
VII - Novara	0,0030448	0,0594	2,24
VIII - Verbania	0,0031756	0,0497	2,33
IX - Cuneo	0,0032094	0,0312	2,31
X - Saluzzo - Savigliano - Fossano	0,0032076	0,0255	2,32
XI - Alba - Bra	0,0031061	0,0242	2,30
XII - Mondovì	0,0030632	0,0180	2,36
XIII - Asti	0,0030049	0,0340	2,22
XIV - Alessandria	0,0029349	0,0826	2,38
XV - Casale Monferrato	0,0028686	0,0206	2,26

CI — INPUT COEFFICIENTS

1. a_{ij}

Industries	1 Propulsive ind.	2 Metals engineering	3 Textiles	4 Clothing	5 Chemicals	6 Food and drink	7 Leather	8 Pulp mills and paper	9 Non met. mining and mineral manuf.	10 Timber Furniture	11 Rubber, tires and cables	12 Poligr. Ind. and Publishing	13 Other manufactures	14 Construction	15 Electricity, gas water	16 Transport and communications
1. Propulsive ind.	0,017422	0,008881												0,000336		0,000508
2. Metals engineering	0,388527	0,410779	0,008214	0,000533	0,010955	0,009197	0,004545	0,027928	0,037086	0,012304	0,064657		0,047522	0,113964	0,118294	0,056083
3. Textiles	0,005053	0,001768	0,280000	0,375096	0,007757	0,000445		0,0033156	0,000117	0,002083	0,116055	0,000684	0,004260		0,000307	
4. Clothing	0,000141		0,000369	0,008223		0,000092	0,026941			0,000062				0,000176		0,000833
5. Chemicals	0,023484	0,019529	0,074224	0,020373	0,434233	0,015790	0,069483	0,138233	0,035954	0,014512	0,085000	0,050000	0,013718	0,019578	0,061850	0,112834
6. Food and drink	0,000235				0,007604	0,160523			0,000029		0,001631					
7. Leather	0,001098	0,002295	0,000567	0,038840		0,000210	0,299913				0,000567		0,000895			
8. Pulp mills and paper	0,002561	0,003425	0,008093	0,004691		0,051019	0,000439	0,277997	0,022756	0,002362	0,041597	0,252848	0,012185	0,000305	0,002099	0,001137
9. Non metallic mining and mineral manufactures	0,014317	0,016395	0,006995		0,010371	0,014250		0,004244	0,197677	0,003930	0,058099		0,001704	0,204060	0,027034	0,000022
10. Timber Furniture	0,003155	0,004607	0,000643	0,000863	0,003761	0,004770		0,006450	0,001302	0,235269			0,005602	0,082927	0,001364	0,000366
11. Rubber, tires and cables	0,048641	0,007062	0,001072	0,007008	0,001553	0,000419	0,001188	0,000689	0,002540	0,003321	0,083634	0,000160	0,002130	0,002351	0,033864	0,034124
12. Polygraphic Ind. and Publishing	0,003311	0,000349	0,001484	0,000107	0,016744	0,007540	0,000569	0,000644	0,003798	0,000072	0,030386	0,085640	0,002599	0,000193	0,007992	0,003658
13. Other manufactures	0,000254	0,005200	0,000914	0,007615	0,003747	0,021715	0,000103	0,007921	0,008180	0,002888	0,004765	0,004874	0,000213	0,001544		0,000116
14. Construction	0,002096	0,004281	0,003686	0,004854	0,004763	0,006428	0,003696	0,002835	0,006292	0,001970	0,003256	0,008695	0,004009		0,061691	0,002288
15. Electricity, gas and water	0,017625	0,018704	0,019366	0,006300	0,018851	0,009431	0,009814	0,030107	0,068682	0,013223	0,026041	0,005993	0,002338	0,009918	0,249188	0,006204
16. Transport and communications	0,015390	0,014015	0,011559	0,013130	0,019000	0,029810	0,008111	0,016808	0,040381	0,017749	0,024642	0,024924	0,006993	0,015336	0,029208	0,023322

C III — COMMERCIAL COEFFICIENTS

1. α_{ij}

Industries	1 Propulsive ind.	2 Metals engineering	3 Textiles	4 Clothing	5 Chemicals	6 Food and drink	7 Leather	8 Pulp mills and paper	9 Non met. mining and mineral manuf.	10 Timber Furniture	11 Rubber, tires and cables	12 Polygraphic Ind. and Publishing	13 Other manufactures	14 Construction	15 Electricity, gas and water	16 Transport and communications
1. Propulsive ind.	1,0000000	1,0000000												1,0000000		1,0000000
2. Metals engineering	0,3492936	0,4252545	0,1903784	0,0806452	0,5850209	0,7582311	0,9034091	0,4306089	0,2714571	0,8382230	0,3158587		0,2545944	0,4355833	0,3291083	0,5857302
3. Textiles	0,4919866	0,8768415	0,2217072	0,7734997	0,3018611	0,8400000		0,2912621	0,8805031	1,0000000	0,1623488	0,1489362	1,0000000		1,0000000	
4. Clothing	1,0000000		0,4509804	0,7562438		0,3225800	0,0201342			0,8333333				0,5370370		1,0000000
5. Chemicals	0,1841440	0,4673037	0,6700243	0,6923387	0,2900000	0,2939445	0,7074349	0,1400000	0,1665604	0,3702914	0,1563955	0,7320261	0,2329193	0,6556017	0,5438797	0,2614321
6. Food and drink	1,0000000				0,2343826	0,3115724					0,3086957					
7. Leather	0,2915952	0,5235540	1,0000000	0,4899358		0,5263158	0,2774955				0,8875000		0,2380952			
8. Pulp mills and paper	0,5967062	0,3679775	0,8728788	0,7740806	0,3143371	0,6667671	0,9411765	0,2509306	0,4795055	0,6812227	0,2006619	0,6751453	0,6573427	0,5212766	0,5219512	0,7963801
10. Non metallic mining and mineral manufactures	0,4167296	0,1858259	0,5136029		0,1779335	0,6924135		0,7292419	0,3200909	1,0000000	0,0349078		1,0000000	0,7859999	1,0000000	1,0000000
10. Timber Furniture	0,9704478	0,8773756	1,0000000	0,5714286	0,5505618	1,0000000		0,0522565	0,8295455	0,4795765			1,0000000	0,4500000	1,0000000	0,5000000
11. Rubber, tires and cables	0,3800000	0,7829938	0,5101124	0,2473623	0,4580499	0,8794326	0,6956522	0,4444444	0,9825073	0,5372671	0,2219568	1,0000000	1,0000000	1,0000000	0,3320788	0,5780883
12. Polygraphic Ind. and Publishing	0,8566553	0,7462687	0,9155844	0,9230769	0,4000000	0,5181782	1,0000000	0,5952381	0,3664717	0,2857143	0,1673279	0,8977408	1,0000000	0,6923077	0,5224072	0,8773333
13. Other manufactures	0,3703704	0,5431034	0,3130435	0,3633005	0,3493976	0,2396969	0,5000000	0,6325581	0,5377049	0,3714266	0,2247024	0,8835821	1,0000000	0,5221053		1,0000000
14. Construction	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000
15. Electricity, gas and water	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	0,4175606	1,0000000
16. Transport and communications	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000	0,7800000

CII — FIXED CAPITAL STOCK COEFFICIENTS

1. b_{ij}

Industries	1 Propulsive ind.	2 Metals engineering	3 Textiles	4 Clothing	5 Chemicals	6 Food and drink	7 Leather	8 Pulp mills and paper	9 Non met. mining mineral manufac.	10 Timber Furniture	11 Rubber, tires cables	12 Polygr. Ind. and Publishing	13 Other manufactures	14 Construction	15 Electricity, gas water	16 Transport and communic.
1. Propulsive ind.	0,0024	0 0261	0,0203	0 0289	0 0261	0,0142	0 0298	0.0263	0.0368	0 0770	0 0263	0,0219	0 0300	0,0607	0,0035	0 7207
2. Metals engineering	0,6309	0 6377	0,7025	0 5293	0 7253	0,2943	0,7154	1,0208	0,8278	0 6600	0 9026	0 5951	0 4458	0,1555	0,4859	1.9087
3. Textiles																
4. Clothing																
5. Chemicals	0,0105	0 0017		0,0168	0,0003			0,0025	0,0309	0 0180						0 0053
6. Food and drink																
7. Leather																
8. Pulp mills and paper																
9. Non metallic mining and mineral manufactures																
10. Timber Furniture	0,0105	0 0017		0 0168	0.0003			0.0025	0 0309	0,0180						0 0053
11. Rubber, tires and cables	0,0105	0 0017		0 0168	0.0003			0,0025	0.0309	0,0180						0 0100
12. Polygraphic Ind. and Publ.																
13. Other manufactures																
14. Construction	0,1430	0,1746	0,1377	0,2123	0,1308	0,1107		0,1979	0,2304	0,2090	0,1678	0,2259	0,1242		0 9105	0,2500
15. Electricity, gas and water																
16. Transport and communicat.																

C III — COMMERCIAL COEFFICIENTS

3. β_{ij}

Industries	1 Propulsive ind.	2 Metals engineering	3 Textiles	4 Clothing	5 Chemicals	6 Food and drink	7 Leather	8 Pulps mills and paper	9 Non met. mining mineral manuf.	10 Timber Furniture	11 Rubber, tires and cables	12 Polygr. Ind. and Publishing	13 Other manufactures	14 Construction	15 Electricity, gas and water	16 Transport and communic.
1. Propulsive ind.	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000
2. Metals engineering	0,300000	0,214853	0,145107	0,180000	0,200000	0 395809	0,129685	0,138127	0,191333	0,336036	0,214019	0,300000	0,345972		0,368126	0 285818
3. Textiles																
4. Clothing																
5. Chemicals	0,385802	0,801369			0 904762				0,158140	0,521595						
6. Food and drink																
7. Leather																
8. Pulp mills and paper																
9. Non metallic mining and mineral manufactures																
10. Timber Furniture	0 385802	0 801369			0 904762				0,158140	0,521595						
11. Rubber, tires and cables	0 385802	0 801369			0 904762				0,158140	0 521595						
12. Polygraphic Ind. and Publ.																
13. Other manufactures																
14. Construction	1,000000	0 928799	0 956084	0 944723	0 997752	0,891609		1,000000	0,935521	1,000000	0,998458	0,717736	1,000000		1,000000	0,955634
15. Electricity, gas and water																
16. Transport and communicat.																

Industries	C I Input coeff. (for tertiary) labour and consumption coeff.			C II Fixed cap. stock coeff. (for tert.)	C III Commercial coefficients				C IV Income coefficients			C VI Population coefficients
	2. a_{is}	3. o_j	6 c_j	2. b_{is}	2. α_{is}	4. β_{is}	5. γ_i	6. e_i	1. s_i	3. π_i	5. μ_i	2. ρ_j
1. Propulsive ind.		9,501408	0,034395	0,0056034		1,0000000	0 850006		0 02356048	0,015000		1,018
2. Metals engineering	0,00041768	14,243047	0,013495	0,0445780	0,578899	0,0576283	0,554629		0 01406212	0,025000	0,0223005	1,120
3. Textiles	0,00002636	22,077672	0,013291		0,590686		0 522689		0 01011057	0,025000	0,0092835	1,030
4. Clotingh	0,00029487	21,013431	0,055000		0,202257		0,600000		0,00878677	0 025000	0 0811190	1,265
5. Chemicals	0,00176844	9,750003	0,041135	0,0019119		0,0501645	0 500000		0,01325559	0 035000	0,0060685	1,035
6. Food and drink	0,00004533	7,727603	0,082691		0,650036		0 522365		0,01237693	0 030000	0,0120041	1,135
7. Leather	0,00003231	11,776913	0,002963				0 522285		0,01120587	0,025000	0 0078576	1,070
8. Pulp mills and paper	0,00033280	15,994962	0 001510				0 537105		0 01209834	0 030000	0,0102834	1,031
9. Non metallic mining and mineral manufactures	0,00010952	16,146088	0 000996		0 252212		1,000000		0 01290358	0 030000	0 0193564	1,090
10. Timber Furniture	0,00013789	19,008688	0 007014	0,0019119	0,582474	0,0501645	0,557569		0 00986258	0 025000	0 0951875	1,365
11. Rubber, tires and cables	0,00007954	9,357470	0 002924	0,0019119	0 784728	0,0501645	0,756154	0,620	0 01686530	0 030000	0,0026047	1,028
12. Polygraphic Ind. and Publ.	0,00086666	13,405121	0 012756		0,096253		0,511274	0,270	0,01660522	0,030000	0 0224276	1,066
13. Other manufactures	0,00069504	8,508506	0 006900		0,026310		0,522094	0 448	0 01282591	0 025000	0 0243730	1,230
14. Construction	0,00080469	19,310332	0 003489	0,0189828	1,000000	1,0000000	1,000000		0,01205724	0,025000	0 0232387	1,170
15. Electricity, gas and water	0,00045199	7,038180	0,019149		1,000000		1,000000		0,02188171			1,018
16. Transport and communicat.	0,00055466	24,920010	0,034002		1,000000		1,000000		0,01368296	0,025000	0,0456410	1,109

CV — COEFFICIENTS FOR ALLOCATION OF ECONOMIC ACTIVITIES AMONG THE ECOLOGICAL AREAS

1. Sn

Zones	1 Propulsive ind.	2 Metals engineering	3 Textiles	4 Clothing	5 Chemicals	6 Food and drink	7 Leather	8 Pulp mills and paper	9 Non met. mining mineral manufac.	10 Timber Furniture	11 Rubber, tires and cables	12 Polygr. Ind. and Publishing	13 Other manufactures	14 Construction	15 Electricity, gas and water	16 Transport and communic.
I - Torino	0,8405	0,6050	0,2708	0,3849	0,3703	0,3062	0,7074	0,4016	0,2571	0,3786	0,7416	0,6956	0,7141	0,4048	0,5231	0,4666
II - Ivrea	0,1118	0,0168	0,0115	0,0110	0,0549	0,0113	0,0244	0,0057	0,0169	0,0157	0,0017	0,0134	0,0051	0,0289	0,0225	0,0135
III - Pinerolo	0,0476	0,0093	0,0240	0,0110	0,0091	0,0159	0,0061	0,0287	0,0358	0,0392	0,0017	0,0066	0,0051	0,0242	0,0169	0,0150
IV - Vercelli		0,0149	0,0161	0,0211	0,0823	0,0340	0,0061	0,0115	0,0142	0,0313	0,0400	0,0133	0,0204	0,0311	0,0255	0,0249
V - Borgosesia		0,0130	0,0978	0,0205	0,0018	0,0091	0,0024	0,1149	0,0380	0,0261	0,0017	0,0066	0,0051	0,0272	0,0099	0,0126
VI - Biella		0,0207	0,3787	0,0422	0,0055	0,0159	0,0488	0,0115	0,0282	0,0261	0,0017	0,0133	0,0102	0,0538	0,0176	0,0311
VII - Novara		0,0656	0,1184	0,1032	0,1406	0,0905	0,0447	0,0542	0,0427	0,0431	0,0075	0,1027	0,0242	0,0762	0,0940	0,0583
VIII - Verbania		0,0539	0,0403	0,0543	0,1700	0,0365	0,0407	0,0655	0,1086	0,0744	0,0018	0,0239	0,0777	0,0789	0,1004	0,0540
IX - Cuneo		0,0130	0,0048	0,0189	0,0091	0,0227	0,0061	0,0102	0,0565	0,0470	0,1000	0,0333	0,0102	0,0378	0,0278	0,0345
X - Saluzzo - Savigliano Fossano		0,0184	0,0097	0,0315	0,0061	0,0453	0,0024	0,0861	0,0335	0,0170	0,0035	0,0200	0,0051	0,0252	0,0221	0,0223
XI - Alba - Bra		0,0104	0,0158	0,0463	0,0210	0,1474	0,0366	0,0115	0,0223	0,0313	0,0070	0,0066	0,0051	0,0261	0,0147	0,0222
XII - Mondovì		0,0110	0,0028	0,0189	0,0123	0,0181	0,0024	0,0345	0,0424	0,0313	0,0005	0,0066	0,0051	0,0222	0,0140	0,0242
XIII - Asti		0,0310	0,0028	0,0551	0,0091	0,1020	0,0024	0,0522	0,0847	0,0653	0,0175	0,0008	0,0104	0,0414	0,0190	0,0482
XIV - Alessandria		0,1034	0,0143	0,1543	0,0988	0,1247	0,0634	0,0532	0,1118	0,1044	0,0703	0,0373	0,0920	0,0950	0,0742	0,1478
XV - Casale Monferrato		0,0136	0,0019	0,0268	0,0091	0,0204	0,0061	0,0287	0,1073	0,0392	0,0035	0,0200	0,0102	0,0272	0,0183	0,0248

C V — COEFFICIENTS FOR THE ALLOCATION OF THE POPULATION WORKING IN ZONE K AMONG THE VARIOUS ECOLOGICAL ZONES

3. v_{hk}

Residence zone \ Employment zone	I Torino	II Ivrea	III Pinerolo	IV Vercelli	V Borgosesia	VI Biella	VII Novara	VIII Verbania	IX Cuneo	X Saluzzo Savigliano Fossano	XI Alba - Bra	XII Mondovì	XIII Asti	XIV Alessandria	XV Casale Monferr
I - Torino	0,975	0,025													
II - Ivrea	0 002	0,967													
III - Pinerolo	0,008		1,000												
IV - Vercelli	0 002	0,005		1,000											
V - Borgosesia					1,000										
VI - Biella		0 003				1,000									
VII - Novara							1,000								
VIII - Verbania								1,000							
IX - Cuneo									1,000						
X - Soluzzo - Savigliano										1,000					
Fossano	0 003														
XI - Alba - Bra	0,004										1,000				
XII - Mondovì												1,000			
XIII - Asti	0,006												1,000		
XIV - Alessandria														1,000	
XV - Casale Monferrato															1,000

9. The temporal features of the model.

It seemed to us that it would not have been worth while troubling ourselves with the construction of a difference equations model to determine the growth of production, employment, consumptions and investment during the period under consideration. This mainly because the solutions we should have obtained would have not been reliable owing to the harmful approximations one could not avoid when computing the eigen values of the system of so many difference equations.

We decided also not to follow the method, suggested by Leontief and Chenery (1), requiring the construction of a chain model which considers separately the productions and employments of each period of the plan. In fact, by following such a procedure, we cannot be sure to be able to solve the system by inverting its matrix of coefficents.

We have then decided to follow an iterative process that will be explained later on. By this method we retrict our analysis to the determination of the values of the relevant variables in the terminal year.

The particular formulation of the model is based on some hypotheses: - the rate of growth of each industrial sector is constant in each year of the planning period. Hence, letting r_i be the rate of growth of the i.th sector, x_i° the present value of the production of the i.th sector, and x_i the value that this production will have in the terminal year, we can write:

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(1) W. Leontief, Input-output economics, New Yorks 1956; H. Chenery, Inter-Industry economics, New York 1959.

$$x_i = (1 + r_i)^T x_i^o$$

where T stands for the number of years of the planning period. Since investments are proportional to the increment of production, they too will increase at the rate r_i per year. Hence the cumulative investments of the i.th capital good by the j.th sector (I_{ij}^T) are given by:

$$I_{ij}^T = \sum_{t=1}^{T} i_{ij}^o (1 + r_j)^t$$

where i_{ij}^o are the corresponding investments in the initial year.

As variables of the model we have the amounts of the final productions. The differences between the final and initial productions give us the increment of productions during the whole T years period. Therefore we can express the cumulative investments as a proportion of such increments:

$$I_{ij}^T = b_{ij} (x_j - x_j^o)$$

The last year's investments (i_{ij}^T) are given by:

$$i_{ij}^T = i_{ij}^o (1 + r_j)^T$$

and they represent the following proportion of the cumulative investments:

$$\varepsilon_j = \frac{i_{ij}^o (1 + r_j)^T}{\Sigma^T i_{ij}^o (1 + r_j)^t}$$

Now, taking notice that:

$$\sum_{t=1}^{T} (1 + r_j)^t = (1 + r_j) \sum_{t=1}^{T} (1 + r_j)^{t-1} = (1 + r_j) \frac{(1 + r_j)^T - 1}{r_i}$$

we can write:

$$\varepsilon_j = \frac{r_j (1 + r_j)^{T-1}}{(1 + r_j)^T - 1}$$

Therefore the investments of the i.th capital good by the j.th industry in the final year, would be:

$$i_{ij}^T = b_{ij}\,\varepsilon_j\,(x_j - x_j^o)$$

Of this amount the proportion β_{ij} comes from industries which operate inside the region. Hence the demand for the i.th good to be invested by the j.th sector and to be provided by the industries of the region is given by:

$$\beta_{ij}\,b_{ij}\,\varepsilon_j\,(x_j - x_j^o)$$

Similar considerations have been made with reference both to the growth of employment in the tertiary sectors and to the growth of non agricultural population which determines the amount of new houses to be built, and to the growth of population in each ecological area on which depends the amount of infrastructures to be provided.

Letting y_s^o and y_s be the employment in the tertiary sectors in the initial and in the terminal year, we have:

$$y_s = y_s^o\,(1 + r_s)^T$$

where r_s stands for the constant rate of growth of the tertiary sectors. By reasoning in a way similar to that done for the industrial sectors, it is possible to prove that the demand for the i.th good by the tertiary sectors for investment purposes, is the following proportion of the cumulative investment of these sectors:

$$\varepsilon_s = \frac{r_s\,(1 + r_s)^{T-1}}{(1 + r_s)^T - 1}$$

Therefore the investment demand for the i.th good by the tertiary sectors to be satisfied inside the region in the terminal year is given by:

$$\beta_{is}\,b_{is}\,\varepsilon_s\,(y_s - y_s^o)$$

The following relationship defines the rate of growth of the non agricultural population of the whole region:

$$\sum_h x_h = (1 + r_c)^T \sum_h p_h^\circ$$

The proportion of the houses to be built during the planning period which has to be imputed in the last year is:

$$c_c = \frac{r_c (1 + r_c)^{T-1}}{(1 + r_c)^T - 1}$$

Therefore the demand for the products of the construction sector in the terminal year is represented by:

$$c_c \, \delta_c \sum_h (x_h - p_h^\circ)$$

The rate of growth of infrastructures in the h.th ecological area is denoted by r_h and is implicitly defined by:

$$x_h + \sigma_h \, o_{ah} = (x_h^\circ + \sigma_h^\circ \, o_{ah}^\circ)(1 + r_h)^T$$

where $\sigma_h \, o_{ah}$ and $\sigma_h^\circ \, o_{ah}^\circ$ denote the agricultural population of the h.th ecological area in the terminal and initial year respectively. The proportion of the cumulative amount of infrastructure to be constructed in the h.th area which has to be imputed in the terminal year is given by:

$$c_h = \frac{r_h (1 + r_h)^{T-1}}{(1 + r_h)^T - 1}$$

Therefore the construction of the infrastructure in the terminal year will generate the following demand for products of the construction sector:

$$\sum_h c_h \, \omega_h \, [x_h - x_h^\circ + \sigma_h \, o_{ah} - \sigma_h^\circ \, o_{ah}^\circ]$$

The expression under square brakets represents the increment of the non agricultural population of the h.th area during the planning period.

10. The equations of the model.

The equations of the model state:

1-3) the production of each industry (for the non autonomous

The equations of the model

1)
$$x_i - \sum_j^{16} \alpha_{ij} a_{ij} x_j - \sum_j^{16} \beta_{ij} b_{ij} \varepsilon_j (x_j - x_j^o) - \alpha_{is} a_{is} y_s -$$
$$- \beta_{is} b_{is} \varepsilon_s (y_s - y_s^o) - \gamma_i c_i x_r = \sum_{l=1}^{7} D_{il}$$

2)
$$(1 - e_i) x_i - \sum_j^{16} \alpha_{ij} a_{ij} x_{ij} - \sum_j^{16} \beta_{ij} b_{ij} \varepsilon_j (x_j - x_j^o) - \alpha_{is} a_{is} y_s -$$
$$- \beta_{is} b_{is} . \varepsilon_s (y_s - y^o_s) - \gamma_i c_i x_r = \sum_{l=1}^{7} D_{il}$$

3)
$$x_{14} - \sum_j^{16} \alpha_{14j} a_{14j} x_{14} - \sum_j^{16} \beta_{14j} \dot{b}_{14j} \varepsilon_{14} (x_j - x_j^o) - \alpha_{14s} a_{14s} y_s -$$
$$- \beta_{14s} \dot{b}_{14s} \varepsilon_s (y_s - y_s^o) \gamma_{14} c_{14} x_r - \varepsilon_e \delta_e \sum_h (z_h - p_h^o) -$$
$$- \sum_h \varepsilon_h \omega_h [z_h - z_h^o + \sigma_h o_{ah} - \sigma_h^o o_{ah}^o] = \sum_l^{6} D_{il}$$

4)
$$- \sigma_h \sum_k \nu_{hk} \sum_j \xi_{jk} \rho_j o_j x_j \quad - \sigma_h \sum_k \nu_{hk} \bar{\zeta}_{sk} \rho_s y_s \quad + z_h$$
$$= \sigma_h \sum_k \nu_{hk} \xi_{ak} \rho_a o_a \qquad h = 1, 2, \dots 15$$

5)
$$- \sum_j^{16} o_s o_j x_j \quad + y_s \quad - o_{sr} x_r \quad = o_t$$

6)
$$- \sum_j^{16} (s_j o_j + \pi_j + \mu_j) x_j \quad - (s_s + \pi_s + \mu_s) y_s$$
$$+ x_r = p_{aa} o_a + s_a o_a + s_s o_s + \sum T_h + T_a + P$$

7)-10)
$$x_j = x_j^o (1 + r_j)^T \qquad y_s = y_s^o (1 + r_s)^T \qquad \sum_h z_h = (1 + r_e)^T \sum_h p_h^o$$
$$z_h + \sigma_h o_a = (z_h^o + \sigma_h^o o_a^o)(1 + r_h)^T \qquad j = 1, 2, \dots 16 \qquad h = 1, 2, \dots 15$$

11)-14)
$$\varepsilon_j = \frac{r_j (1 + r_j)^{T-1}}{(1 + r_j)^T - 1} \quad j = 1, 2 \dots 16 \qquad \varepsilon_s = \frac{r_s (1 + r_s)^{T-1}}{(1 + r_s)^T - 1}$$
$$\varepsilon_e = \frac{r_e (1 + r_e)^{T-1}}{(1 + r_e)^T - 1} \qquad \varepsilon_h = \frac{r_h (1 + r_h)^{T-1}}{(1 + r_h)^T - 1} \quad h = 1, 2 \dots 15$$

industries the production not exported outside the region) must be equal to the quantities demanded as intermediate products and as a capital goods (for the quota charged on the last year) by the industries and the tertiary sectors, plus the quantities consumed by the households plus the exogeneous demand (for the autonomous industries the exogeneous demand includes also the exports outside the region). For the constru ction industry the demand for houses and for social capital has to be added

4) the population in each ecological area depends on the population working in the various areas and resident in the ecological area. The population working in the various areas depends on the coefficients for the allocations of the industries and tertiary among the different areas

5) the employment in the tertiary sectors depends on the variables which we have already recalled

6) households' income is made up of the components already recalled

7) the definition of the rate increase in the industrial production for each industry

8) the definition of the rate of increase in the employment of the tertiary sectors

9) the definition of the rate of increase in the non agricultural population living in houses not to be rebuilt

10) the definition of the rate of increase in the population of each ecological area

11-14) the quotas of investments, of the value of the houses to be built and of the infrastructures to be produced at the terminal year.

11. The solution of the model

The model is not linear. To solve it we have single out the Leontief part of the model by transferring other terms including unknowns to the other side of the equations (the side of the predetermined values). We have then applied a process of iteration starting from an ex ante set of values of the various unknowns: the process has come out to be convergent and has stopped when differences between the ex ante and ex post values of the unknowns (resulting from the solution of the model), not larger than 3% have been reached.

An iterative process has assured the equalisation of the ex ante evaluations of the region's demand for the products of the autonomous industries and the ex post computations provided by the solution of the model.

The computation procedure has been planned in such a way that all explorations of alternative sets of guesses can be easily done: their economic, spatial and financial implications can be obtained very quickly.

The solutions for the set of guesses which did result to be the most reasonable are given in the following tables.

TAB. 1

THE SOLUTION OF THE MODEL - PRODUCTION AND EMPLOYMENT

Industries	Production at 1970	Yearly rate of growth	Employment at 1970	Yearly rate of growth
1. Propulsive ind.	19.440	9,0	184.706	3,30
2 Metals engineering	17.445	8,25	264.618	2,15
3 Textiles	5.681	4,0	121.452	—1,23
4. Clothing	2.326	9,5	58.650	1,90
5. Chemicals	5.731	10,3	56.720	4,00
6. Food and drink	5.169	6,20	43.135	1,70
7. Leather	607	5,5	7.511	0,25
8. Pulp mills and paper	1.132	7,4	17.233	1,25
9. Non'metallic mining and mineral manuf.	2.444	8,3	40.846	2,10
10. Timber Furniture	1.280	4,0	31.637	—0,77
11. Rubber, tires and cables	2.348	6,5	22.187	1,35
12. Polygraphic Ind. and Publishing	1.314	9,0	17.474	3,35
13. Other manufactures	712	6,15	7.800	1,10
14. Construction	7.996	9,65	162.122	3,75
15. Electricity, gas and water	2.609	10,2	15.600	2,45
16. Transport and communications	3.280	4,0	89.099	0,90

TAB. 2

THE SOLUTION OF THE MODEL - INVESTMENTS

	Investments at 1970	Investments over the period 1964-'70
1. Propulsive ind.	1298	7121
2. Metals engineering	1115	6267
3. Textiles	190	1189
4. Clothing	165	900
5. Chemicals	474	2526
6. Food and drink	125	738
7. Leather	23	140
8. Pulp mills and paper	98	564
9. Non metallic mining and mineral manufactures	214	1223
10. Timber Furniture	49	311
11. Rubber, tires and cables	154	913
12. Polygraphic Ind. and Publishing	92	508
13. Other manufactures	25	148
14. Construction	150	821
15. Electricity, gas and water	324	1757
16. Transport and communications	359	2274

TAB. 3

THE SOLUTION OF THE MODEL

TOTAL POPULATION AND ACTIVE POPULATION RESIDING IN THE VARIOUS ZONES

Zones	Total population		Perc. Change 1963-1970	Number of non agricultural workers		Perc. Change 1963-1970	perc. employment in non agricultural sectors		perc. of active popol. on total pop.	
	1963	1970		1963	1970		1963	1970	1963	1970
I - Torino	1.775,3	2.044,7	15,1	743,9	854,6	14,9	92,7	94,8	44,9	44,3
II - Ivrea	109,1	134,9	23,6	40,2	50,7	26,1	75,7	83,7	47,5	45,9
III - Pinerolo	113,2	124,2	9,7	31,4	36,0	14,6	69,3	76,8	45,6	44,3
IV - Vercelli	124,4	132,8	6,7	31,9	39,7	24,5	61,7	71,4	45,4	44,7
V - Borgosesia	81,7	82,5	1,0	30,6	34,4	12,4	86,0	89,6	48,5	48,0
VI - Biella	184,8	201,2	8,9	83,9	88,9	6,0	92,4	92,7	50,3	48,8
VII - Novara	258,4	291,9	11,3	88,6	106,2	19,9	81,4	87,7	45,9	44,6
VIII - Verbania	201,0	228,0	13,4	74,8	85,9	14,8	91,0	93,5	44,0	42,9
IX - Cuneo	139,9	145,3	3,9	33,6	40,7	21,1	57,5	67,3	44,4	43,3
X - Saluzzo - Savigliano - Fossano	137,1	141,8	3,4	27,9	34,0	21,9	50,1	59,9	43,5	43,0
XI - Alba - Bra	128,0	135,5	5,8	27,1	35,0	29,2	51,2	64,9	44,1	43,7
XII - Mondovì	97,4	97,9	0,5	19,9	24,6	23,6	50,1	62,3	43,8	42,4
XIII - Asti	198,8	204,8	3,0	43,0	52,6	22,3	52,0	62,3	45,9	44,9
XIV - Alessandria	394,9	417,5	5,7	112,9	133,7	18,4	69,3	78,9	42,9	42,2
XV - Casale Monferrato	108,0	108,0	—	25,3	30,6	20,9	56,7	67,3	45,3	44,4
Total	4.052,0	4.491,0	10,8	1.415,0	1.647,6	16,4	80,1	85,8	45,1	44,3