WORKING PAPERS

A LARGE SCALE MODEL FOR TURIN METROPOLITAN AREA

C. S. Bertuglia, S. Occelli, G.A. Rabino,
C. Salomone, R. Tadei

WP n. 3

ires ISTITUTO RICERCHE ECONOMICO - SOCIALI DEL PIEMONTE

Working Paper n. 3

A LARGE SCALE MODEL FOR TURIN METROPOLITAN AREA

C. S. Bertuglia, S. Occelli, G.A. Rabino,

C. Salomone, R. Tadei

May 1981

Paper presented at the Italo-Swedish Conference on Regional Economics, Preganziol, Italy, 5-7/5/1981

CONTENTS

1. Introduction

This work describes a mathematical simulation model(+) of the evolution of the structure of the urban system of Turin.

The socio-economic aspects of the theoretical structure of the model are based on the causal diagram of the Lowry model (Lowry,1964) and the territorial aspects on Wilson's entropy-maximizing method (Wilson,1970); the mathematical formalization of the dynamic aspects, is inspired by the Forrester model (Forrester,1969).

2. The system and the simulation model

2.1. The theoretical framework

2.1.1. Introduction

The system under consideration has been divided into interactive subsystems. The subsystems taken into consideration are:

1. the industrial subsystem;
2. the population subsystem;
3. the service subsystem;
4. the residential subsystem;
5. the transport subsystem;
6. the land use subsystem;
7. housing subsystem.

2.1.2. The industrial subsystem

The industrial subsystem, as in the causal diagram of the Lowry model and, more generally, in the urban economic base theory, is considered the basic subsystem (of the economic growth of the urban system).

The following phenomena (and relative policies) are simulated: the

(+) This is an updated version of the model presented in part (theoretical aspects)in Bertuglia, Occelli, Rabino, Tadei (1980) and in part (operational aspects) in Bertuglia, Occelli, Rabino, Salomone, Tadei (1981), with some substantial differencesdue to operational improvement of the model itself.

building of new industrial plants, the closing of existing industrial plants, the relocation of existing industrial plants, and the variation of jobs in the industrial plants already in existence.

The industrial subsystem is illustrated in fig. 1.

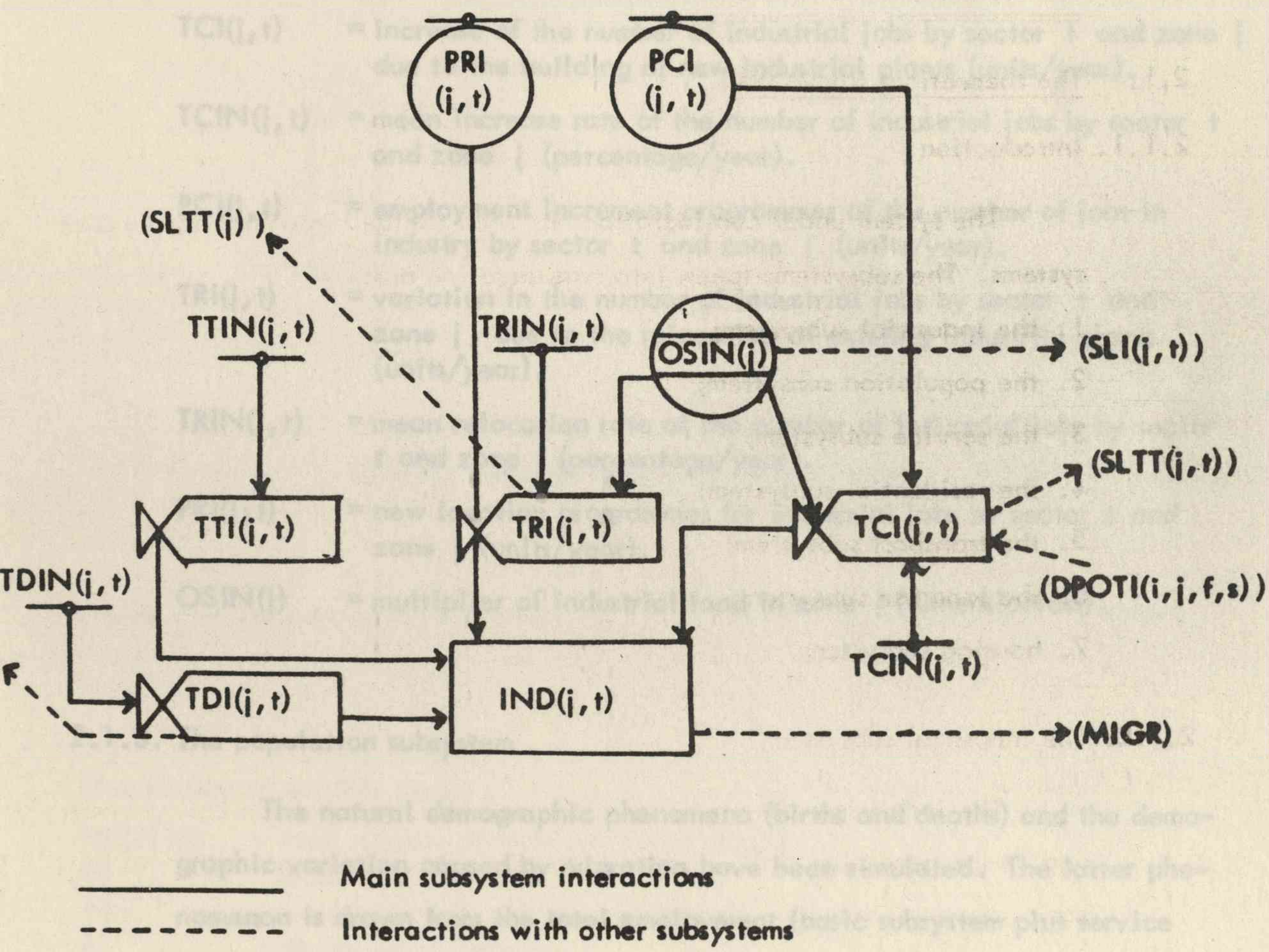


Figure 1 - The industrial subsystem

Description of symbols used:

IND(j,t) = number of industrial jobs by sector t and zone j (units).

TTI(j,t) = variation of the number of industrial jobs by sector t and zone j due to the increase or diminution of jobs in existing factories (units/year).

TTIN(j,t) = mean variation rate of the number of industrial jobs by sector t and zone j (percentage/year).

TDI(j,t) = diminution of the number of industrial jobs by sector t and zone j due to the closing down of factories (units/year).

TDIN(j,t) = mean diminution rate of the number of industrial jobs by sector t and zone j (percentage/year).

TCI(j,t) = increase of the number of industrial jobs by sector t and zone j due to the building of new industrial plants (units/year).

TCIN(j,t) = mean increase rate of the number of industrial jobs by sector t and zone j (percentage/year).

PCI(j,t) = employment increment programmes of the number of jobs in industry by sector t and zone j (units/year).

TRI(j,t) = variation in the number of industrial jobs by sector t and zone j, due to the relocation of existing industrial plants (units/year).

TRIN(j,t) = mean relocation rate of the number of industrial jobs by sector t and zone j (percentage/year).

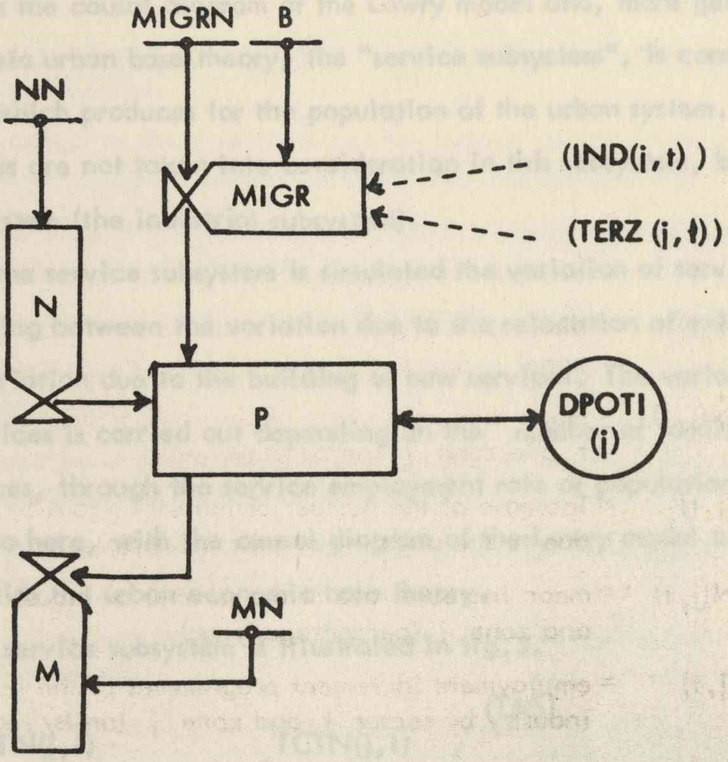
PRI(j,t) = new location programmes for industrial jobs by sector t and zone j (units/year).

OSIN(j) = multiplier of industrial land in zone j (dimensionless).

2.1.3. The population subsystem

The natural demographic phenomena (births and deaths) and the demographic variation caused by migration have been simulated. The latter phenomenon is drawn from the total employment (basic subsystem plus service subsystem) of the urban system, by the rate of employment of the population, in accordance with the economic urban base theory.

The population subsystem is illustrated in fig. 2.



——— Main subsystem interactions

- - - - - - - Interactions with other subsystems

Figure 2 - The population subsystem

Description of symbols used:

DPOTI(j)	= total number of families by workplace j.
N	= births (units/year).
NN	= mean birth rate (percentage/year).
M	= deaths (units/year).
MN	= mean death rate (percentage/year).
MIGR	= migrants (units/year).
MIGRN	= mean migration rate (percentage/year).
B	= mean employment rate (percentage/year).

2.1.4. The service subsystem

As in the causal diagram of the Lowry model and, more generally, in the economic urban base theory, the "service subsystem", is considered as the subsystem which produces for the population of the urban system. So the superior services are not taken into consideration in this subsystem, but in the basic subsystem (the industrial subsystem).

For the service subsystem is simulated the variation of service jobs distinguishing between the variation due to the relocation of existing services and the variation due to the building of new services. The variation of jobs in the services is carried out depending on the number of families who require such services, through the service employment rate of population, in accordance, also here, with the causal diagram of the Lowry model and, more generally, with the urban economic base theory.

The service subsystem is illustrated in fig.3.

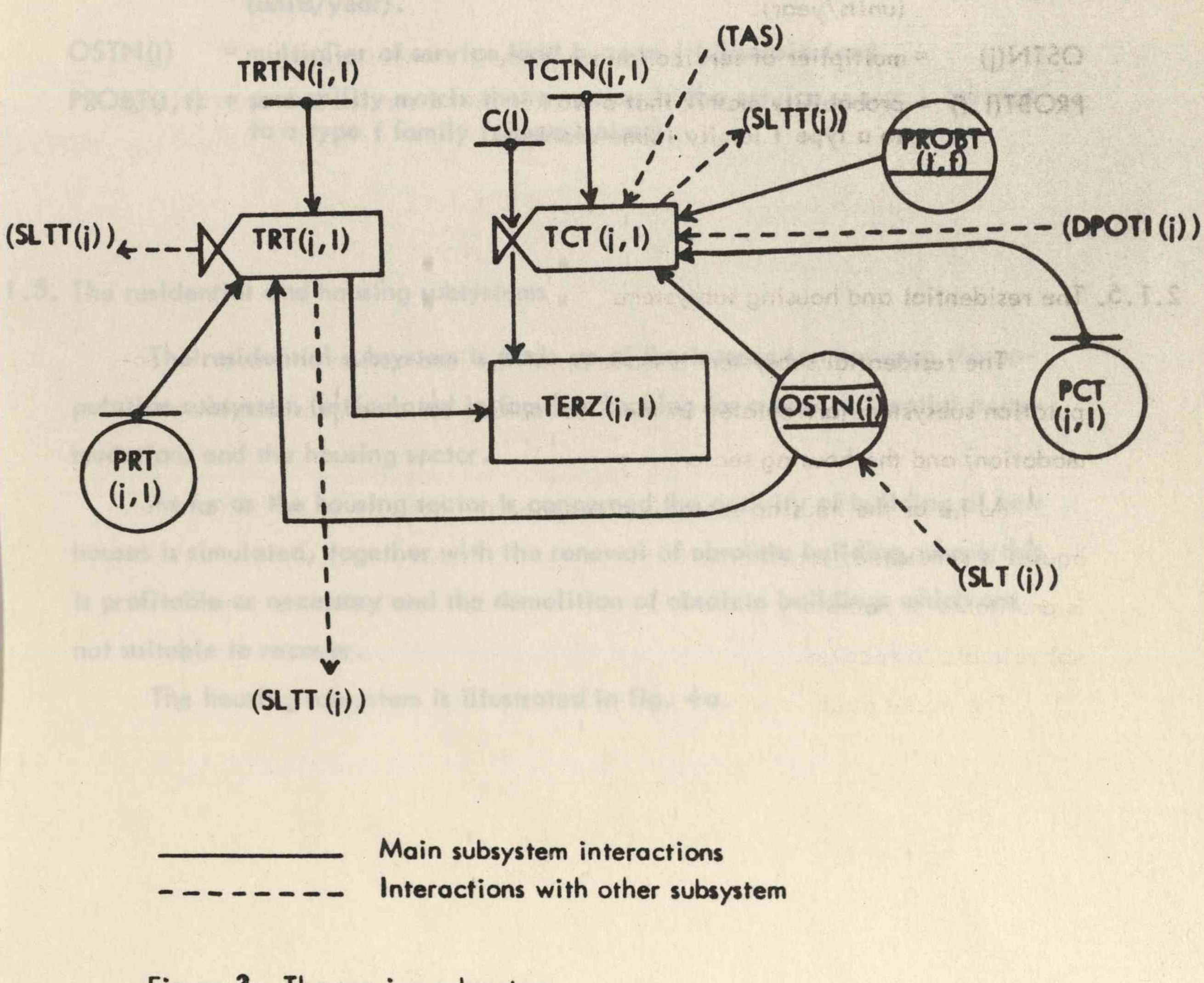


Figure 3 - The service subsystem

Description of symbols used:

TERZ(j,l) = number of workers employed in service jobs by sector l and zone j (units).

TCT(j,l) = variation in the number of service jobs by sector l and zone j due to any phenomenon different from the relocation.

TCTN(j,l) = mean rate of variation in the number of service jobs by sector l and zone j (units/year).

C(l) = mean service employment rate by sector l (percentage/year).

PCT(j,l) = programmes for job increment in services by sector l and zone j (units/year).

TRT(j,l) = variation in the number of service jobs by sector l and zone j, due to relocating services (units/year).

TRTN(j,l) = mean rate of variation of relocating service jobs by sector l and zone j (percentage/year).

PRT(j,l) = new locating programmes for service jobs by sector j and zone j (units/year).

OSTN(j) = multiplier of service land in zone j (dimensionless).

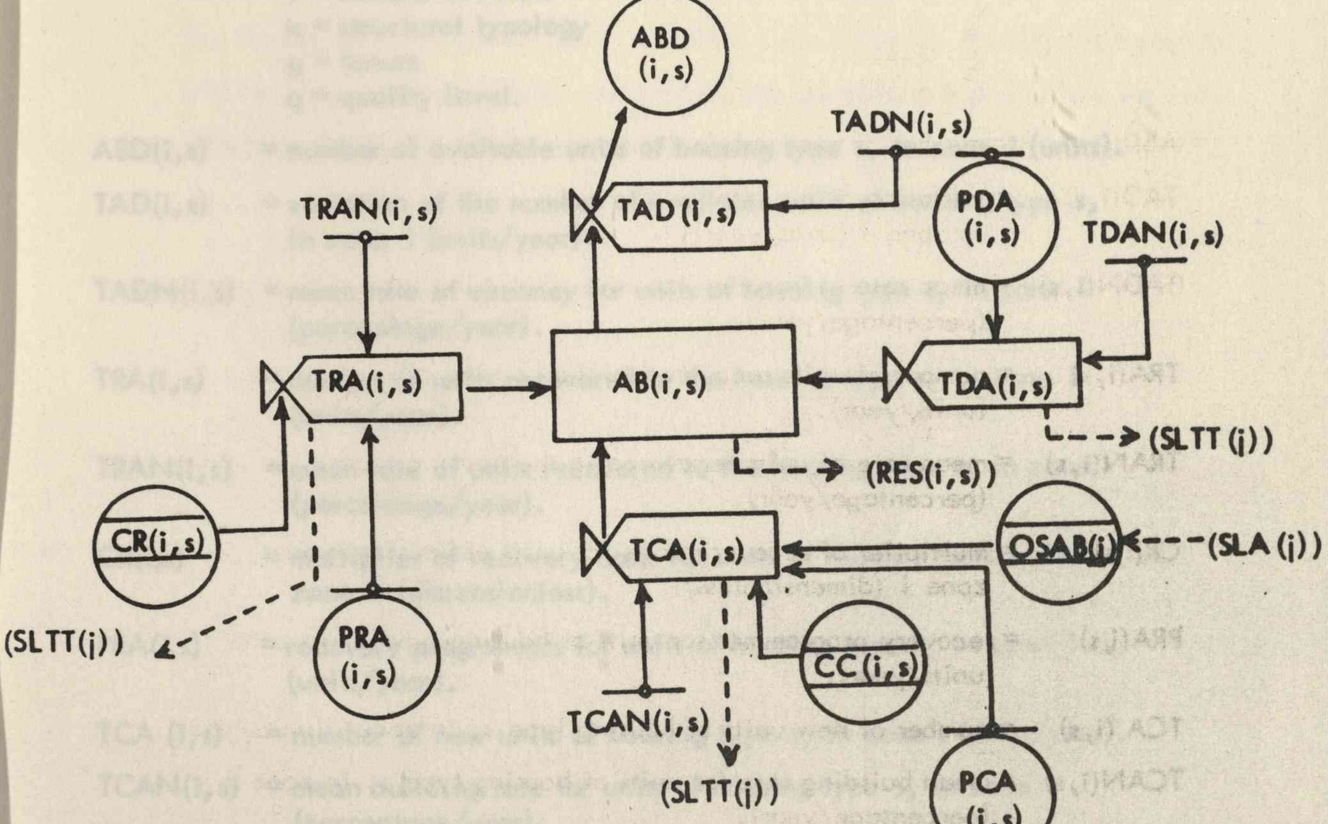
PROBT(l,f) = probability matrix that a worker in the service sector l belongs to a type f family (dimensionless).

2.1.5. The residential and housing subsystems

The residential subsystem is made up of the interaction between the population subsystem (articulated in families looking for a new residential accomodation) and the housing sector.

As far as the housing sector is concerned the activity of building of new houses is simulated, together with the renewal of obsolete building, where this is profitable or necessary and the demolition of obsolete buildings which are not suitable to recover.

The housing subsystem is illustrated in fig. 4a.



——————— Main subsystem interactions

- - - - - - - Interactions with other subsystems

Figure 4a - The housing subsystem

Description of symbols used:

AB(i,s) = number of existing units of housing type s, in zone i (units), where s identifies the different submarket of the housing sector:
s = (v,k,g,q)
v = number of rooms
k = structural typology
g = tenure
q = quality level.

ABD(i,s) = number of available units of housing type s, in zone i (units).

TAD(i,s) = variation of the number of available units of housing type s, in zone i (units/year).

TADN(i,s) = mean rate of vacancy for units of housing type s, in zone i (percentage/year).

TRA(i,s) = number of units recovered to the housing type s, in zone i (units/year).

TRAN(i,s) = mean rate of units recovered to the housing type s, in zone i (percentage/year).

CR(i,s) = multiplier of recovery costs for units of housing type s, in zone i (dimensionless).

PRA(i,s) = recovery programmes for units of housing type s, in zone i (units/year).

TCA (i,s) = number of new units of housing type s, in zone i (units/year).

TCAN(i,s) = mean building rate for units of housing type s, in zone i (percentage/year).

CC(i,s) = multiplier of building costs for units of housing type s, in zone i (dimensionless).

PCA(i,s) = building programmes for units of housing type s, in zone i (units/year).

OSAB(j) = multiplier of residential land s, in zone j (dimensionless).

TDA(i,s) = number of units of housing type s demolished, in zone i (units/year).

TDAN(i,s) = mean demolition rate of units of housing type s, in zone i (percentage/year).

PDA(i,s) = demolition programmes for units of housing type s in zone i (units/year).

As far as the residential subsystem is concerned, the process of residential location of the families is simulated. Such a process is conditioned by the availability of houses, besides the quantity and location of the variables which represent each subsystem of the urban system. In this sense, the simulation of the residential location process constitutes the heart of the simulating model and for this reason it is dealt with elsewhere (section 2.2.3.). Here we only illustrate in fig. 4b the residential subsystem.

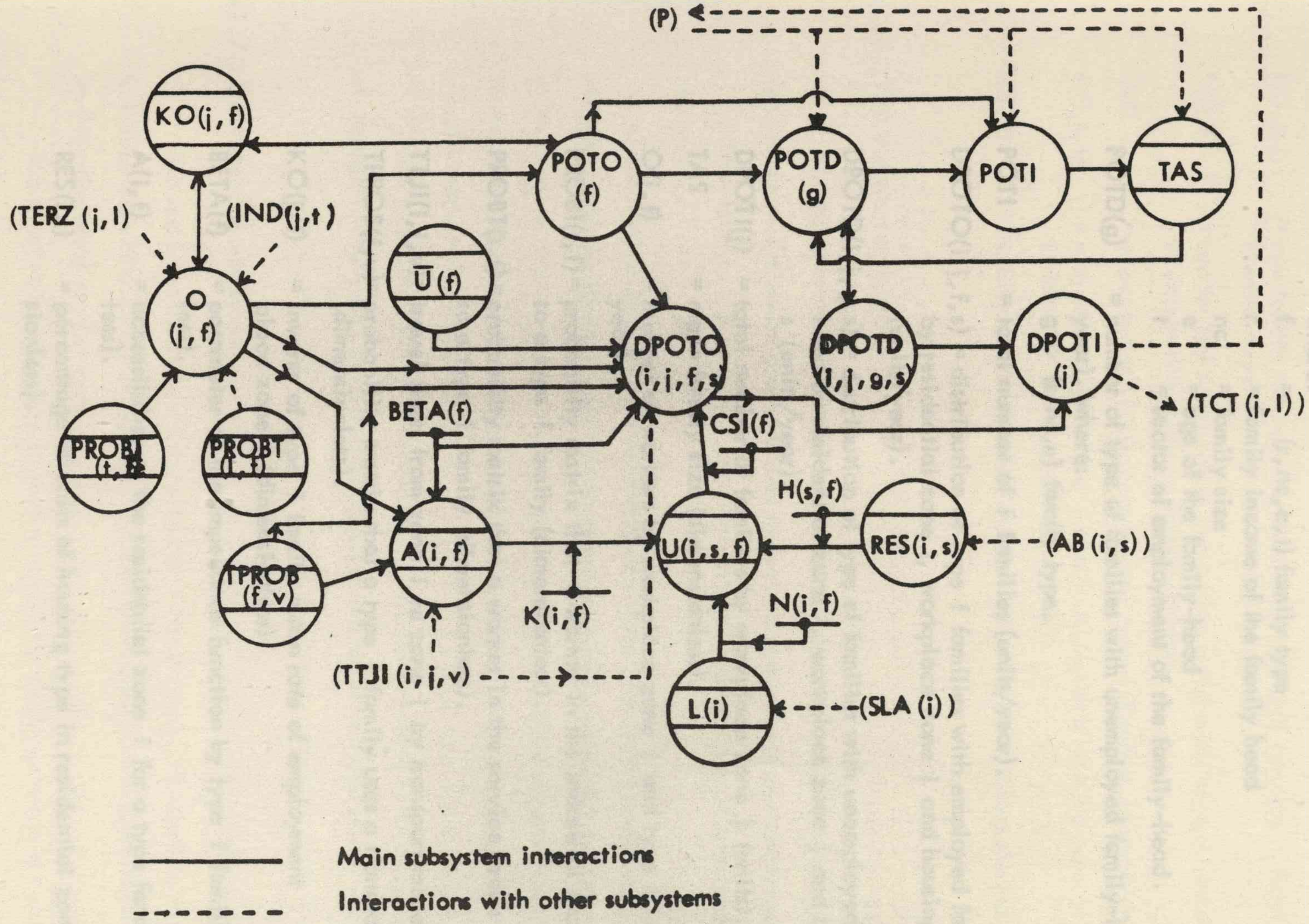


Figure 4b - The residential subsystem

Description of symbols used:

POTO(f) = number of type f families with employed family-head (units/year), where:

f = (r, nc, e, t) family type
r = family income of the family head
nc = family size
e = age of the family-head
t = sector of employment of the family-head.

POTD(g) = number of type of families with unemployed family-head (units/year), where:
g= (r, nc, e) family type.

POTI = total number of f families (units/year).

DPOTO(i, j, f, s) = distribution of type f families with employed family-head by residential zone i, workplace zone j and housing type s (units/year).

DPOTD(i, j, g, s) = distribution of type of families with unemployed family-head by residential zone i, workplace zone j and housing type s (units/year).

DPOTI(j) = total number of families by workplace zone j (units).

TAS = mean family size (dimensionless).

O(j, f) = number of workers by workplace zone j and type f family (units/year).

PROBI(t, f) = probability matrix that a worker in the industrial sector t belongs to a type f family (dimensionless).

PROBT(l, f) = probability matrix that a worker in the service sector l belongs to a type f family (dimensionless).

TTJI(i, j, v) = travel times from zone i to zone j by transport mode v (minutes).

TPROB(f, v) = probability matrix that a type f family uses a transport mode v (dimensionless).

KO(j, f) = inverse of type f family mean rate of employment by workplace zone j (dimensionless).

BETA(f) = parameter of the impedance function by type f families (dimensionless).

A(i, f) = accessibility of the residential zone i for a type family (dimensionless).

RES(i, s) = percentage of units of housing type in residential zone i (dimensionless).

L(i) = percentage of residential land in zone i (dimensionless).

K(i, f) = accessibility evaluation weights for a type f family in zone i (dimensionless).

H(i, s, f,) = residential availability evaluation weights for type f families housing type s in zone i (dimensionless).

N(i, f) = residential attractivity evaluation weights for type f families of residential zone i (dimensionless).

$\overline{U}$(f) = expected utility for the overall type f families (dimensionless).

U(i, s, f) = real utility for a type f family living in a housing type s in residential zone i (dimensionless).

CSI(f) = utility evaluation weights for type f families (dimensionless).

2.1.6. The transport subsystem

The transport subsystem describes the relative accessibilities of the different zones of the urban area, through the values of travel times between zones.

Obviously, travel times vary according to the number of people using the routes.

The number of travellers on the routes varies, in its turn, according to the distributions of housing and employment, but these distribution, are determined by the model. So we recognize a feedback loop: from the travel times to the distribution of workplaces and houses to the travelling times conditioned by these distributions.

The above mentioned phenomena is analized by using a suitable model of transport which will be illustrated further on (section 2.2.2.).

2.1.7. The land-use subsystem

The land-use subsystem intervenes as a conditioning element to the building activities in the industrial, service and residential subsystems and to the relocation programmes.

The availability of land in each zone of the urban system is a function of the building and demolition activities that are undertaken in that zone, in addition to the industrial and service relocation programmes applied.

On the other hand, the spatial distribution of available land contributes to determine for each subsystem the relative attraction, for location, of the different zones for each subsystem.

The land-use subsystem is illustrated in fig. 5.

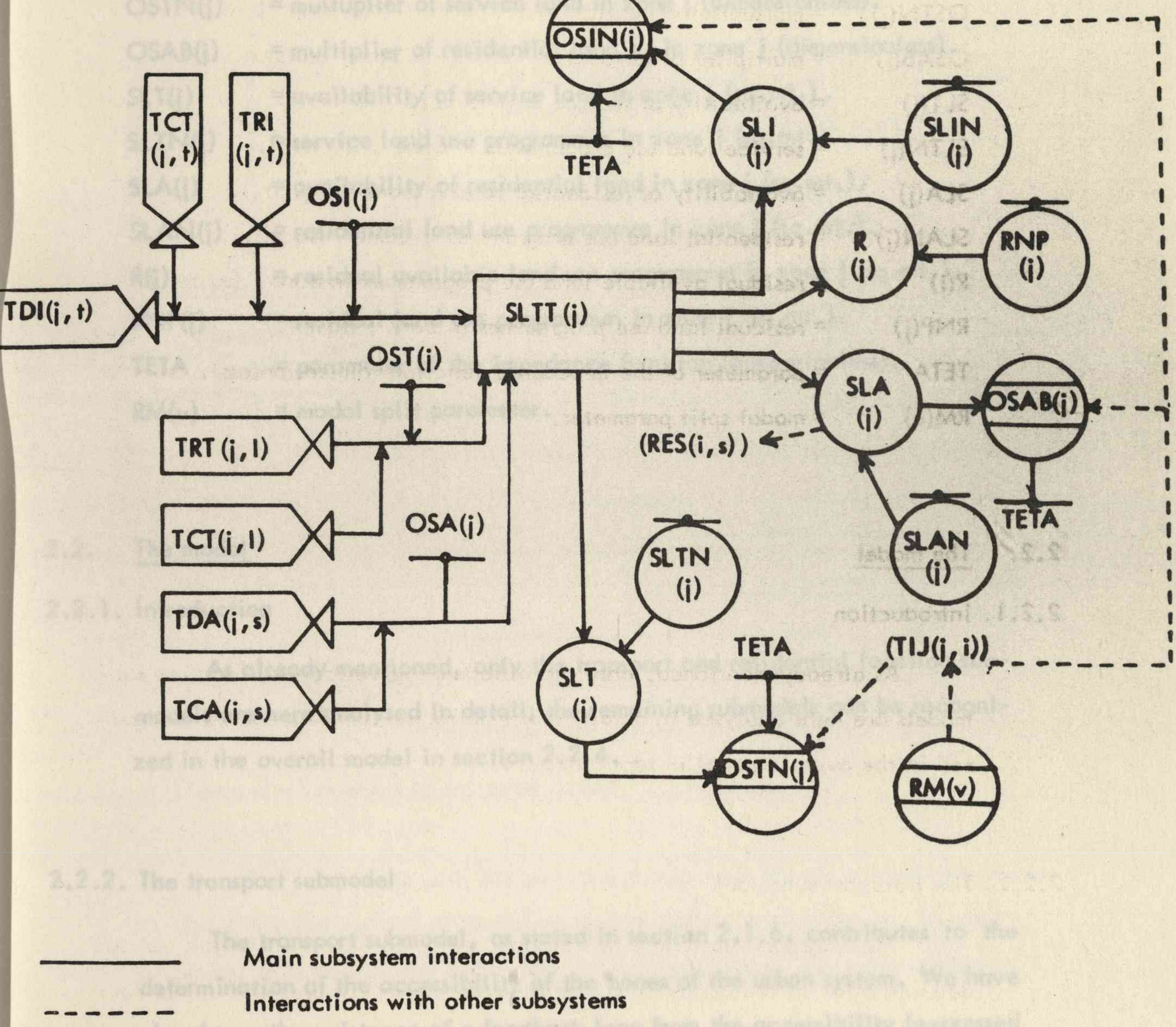


Figure 5 - The land-use subsystem

Description of symbols used:

SLTT(j) = available land in zone j (sq.mt.).

OSI(j) = land occupied by an industrial workplace in zone j (sq.mt.).

OST(j) = land occupied by a service workplace in zone j (sq.mt.).

OSA(j) = land occupied by a family for residential use in zone j (sq.mt.).

SLI (j) = availability of industrial land in zone j (sq.mt.).

SLIN(j) = industrial land use programmes in zone j (sq.mt.).

OSIN(j) = multiplier of industrial land in zone j (dimensionless).

OSTN(j) = multuplier of service land in zone j (dimensionless).

OSAB(j) = multiplier of residential land s, in zone j (dimensionless).

SLT(j) = availability of service land in zone j (sq.mt.) .

SLTN(j) = service land use programmes in zone j (sq.mt.).

SLA(j) = availability of residential land in zone j (sq.mt.).

SLAN(j) = residential land use programmes in zone j (sq.mt.).

R(j) = residual available land use programmes in zone j (sq.mt.).

RNP(j) = residual land use programmes in zone j (sq.mt.).

TETA = parameter of the impedance function (dimensionless).

RM(v) = modal split parameter.

2.2. The model

2.2.1. Introduction

As already mentioned, only the transport and residential location submodels are here analysed in detail; the remaining submodels can be recognized in the overall model in section 2.2.4. .

2.2.2. The transport submodel

The transport submodel, as stated in section 2.1.6. contributes to the determination of the accessibility of the zones of the urban system. We have also shown the existence of a feedback loop from the accessibility (expressed in function of the travel times) to the distribution of workplaces and houses, to the accessibility conditioned by this distribution.

The transport model presented here which takes into account the above mentioned interactions.

The structure of the model is shown in fig. 6.

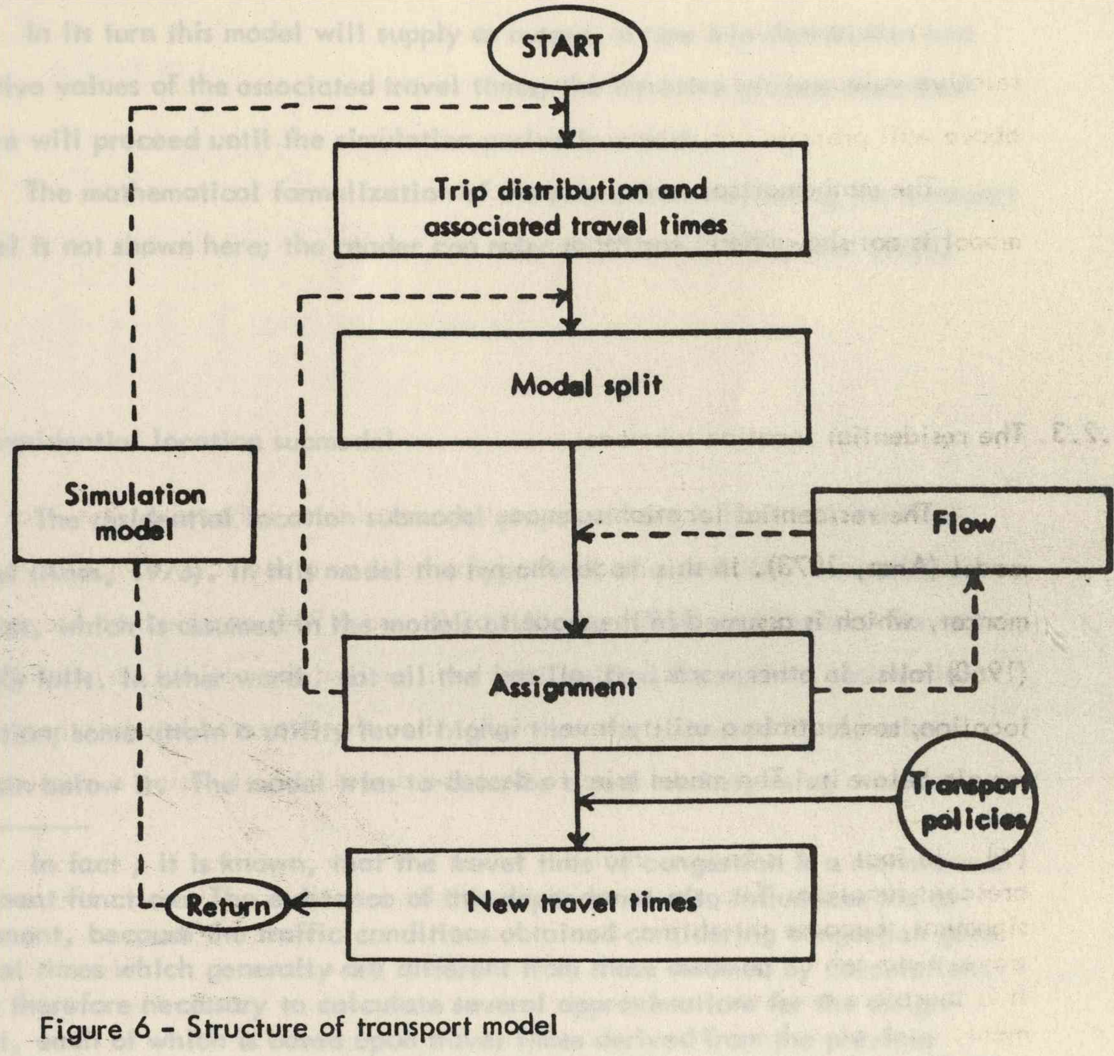


Figure 6 - Structure of transport model

The loop starts with the trip distribution and the associated time values which are recognized at the beginning of the simulation period. In other words, it is known, for every pair origin-destination, the total number of journeys that the members of each family make to go to work and to the services and the average time employed. The loop continues with the modal split submodel, which determines the distribution for the transport modes. Then, the assignation submodel determines the total flow of traffic which charges each link of the transport network. Finally the flow submodel determines the travel time for each link of the network in function of the traffic flow which charges it (+).

In its turn this model will supply as output, a new trip distribution and relative values of the associated travel times; the iterative process described above will proceed until the simulation period is ended.

The mathematical formalization of the submodel constituting the transport model is not shown here; the reader can refer to Wilson (1974), Ires (1978).

2.2.3. The residential location submodel

The residential location submodel proposed here is based on the Anas model (Anas, 1973). In this model the hypothesis of a perfectly competitive market, which is assumed in the models of Alonso (1964) and Herbert-Stevens (1960) falls. In other words, not all the families find the optimal residential location; some attain a utility level higher than the expected level, others remain below it. The model tries to describe a real market, using a measure

---

(+) In fact, it is known, that the travel time vs congestion is a monotonous crescent function. The existence of this dependence also influences the assignment, because the traffic conditions obtained considering congestion give travel times which generally are different from those assumed by calculation. It is therefore necessary to calculate several approximations for the assignment, each of which is based upon travel times derived from the previous one. This process converges to an equilibrium solution, which can be assumed as the final assignment. To this assignation will be associated travel times, generally different from the initial travel times (that is, the ones assumed at the beginning of the simulation period). These new times modify the modal split in an iterative process which is repeated until the desired convergence is reached, forming the definite travel times.

of disequilibrium based on the difference between the expected utility and the real utility for the residential location of the families.

Our model tries to find the most probable residential location by entropy maximization:

$$\max S = - \sum_i \sum_j \sum_f \sum_s \ln DPOTO^f_{ijs}! \quad (1)$$

subject to

$$\sum_i \sum_s DPOTO^f_{ijs} = Q^f_j \qquad j = 1,2,\ldots\ldots J; \; f=1,2\ldots\ldots,F \quad (2)$$

$$\sum_i \sum_j \sum_s \sum_v DPOTO^f_{ijs} \cdot TPROB^f_v \cdot t^v_{ij} = T^f \quad f=1,2\ldots\ldots,F \quad (3)$$

$$\sum_i \sum_j \sum_s DPOTO^f_{ijs} \; (\bar{U}^f - U^f_{is}) = \Delta U^f \qquad f=1,2\ldots\ldots,F, \quad (4)$$

where:

$DPOTO^f_{ijs}$ is the number of type f families with employed family-head by residential zone i, workplace zone j and housing type s;

Q^f_j is the number of families by workplace zone j and type f family;

T^f is the total cost of travels for the type f families;

$TPROB^f_v$ is the probability that a type f family uses a transport mode v;

t^v_{ij} is the travel cost from zone i to zone j by mode v ;

$\bar{U}^f$ is the expected utility value for the overall type f families;

U^f_{is} is the real utility value for type f families living in a housing type s in residential zone i.

The solution of the system (1), (2), (3), (4) is :

$$DPOTO^f_{ijs} = \sum_v B^f_i Q^f_i TPROB^f_v \exp(-\beta^f t^v_{ij}) \exp\left\{ \xi^f [\bar{U}^f - U^f_{is}] \right\}, \qquad (5)$$

where:

$$B^f_i = \left(\sum_i \sum_s \sum_v TPROB^f_v \exp(-\beta^f t^v_{ij}) \exp\left\{ - \xi^f [\bar{U}^f - U^f_{is}] \right\} \right)^{-1} .$$

As far as concerns the utility function, our residential location submodel presents two features:

a. as in the Anas model, the transport costs (that is the travel costs) to travel from the origin i to destination j, do not appear in the utility function and must be subjected to the constraint on the total travel time;

b. the utility function of the families is defined through the fuzzy subsets theory. In other words, the level of family's utility (that is, the level of satisfaction that a type f family would reach living in housing type s in zone i) would be affected by a certain imprecision due, for example, to the difficulty for the family in discriminating between the different advantages (Ponsard, 1978). The utility of the families is configured as a function with several variables evaluated according to subjective criteria. Further aspects of this problem will be discussed in section 3.

2.4. The overall model

We give below the complete mathematical formulation of the model (#).

Industrial submodel

# IND . X(j,t)	= INDN(j,t)
TCI.XY(j,t)	= IND.X(j,t) * TCIN(j,t) * OSIN.XY(j) + PCI.X(j,t)
TRI.XY(j,t)	= IND.X(j,t) * TRIN(j,t) * OSIN.XY(j) + PRI.X(j,t)
TTI.XY(j,t)	= IND.X(j,t) * TTIN(j,t)
TDI.XY(j,t)	= IND.X(j,t) * TDIN(j,t)
IND.Y(j,t)	= IND.X(j,t)+(DT) * (TCI.XY(j,t)+TRI.XY(j,t)+TTI.XY(j,t) - - TDI.XY(j,t))
IND.Y	= $\sum_j \sum_t$ IND.Y(j,t)

(#) All the operations marked by # are the initial statements of the submodels.

Service submodel

$$\# TERZ \cdot X(j,l) = TERZN(j,l)$$

$$TCT \cdot XY(j,l) = \left\{ \left[C(l) * DPOTI \cdot X(j) * TAS \cdot X \right] - TERZ \cdot X(j,l) \right\} * TCTN(j,l) * OSTN \cdot XY(j) + PCT \cdot X(j,l)$$

$$TRT \cdot XY(j,l) = TERZ \cdot X(j,l) * TRTN(j,l) * OSTN \cdot XY(j) + PRT \cdot X(j,l)$$

$$TERZ \cdot Y(j,l) = TERZ \cdot X(j,l) + (DT) * \left(TCT \cdot XY(j,l) + TRT \cdot XY(j,l) \right)$$

$$TERZ \cdot Y = \sum_j \sum_l TERZ \cdot Y(j,l)$$

Population submodel

$$\# P,X = PN$$

$$N \cdot XY = P.X. * NN$$

$$M.XY = P.X. * MN$$

$$MIGR.XY = \left[(IND.X + TERZ.X) * 1/B - P.X \right] * MIGRN$$

$$P.Y. = P.X. + (DT) * (N.XY - M.XY + MIGR.XY)$$

Housing submodel

$$\# AB.X(i,s) = ABN(i,s)$$

$$TRA.XY(i,s) = AB.X(i,s) * TRAN(i,s) * CR(i,s) + PRA.X(i,s)$$

$$TCA.XY(i,s) = AB.X(i,s) * TCAN(i,s) * CC(i,s) * OSAB.XY(j) + PCA.X(i,s)$$

$$TDA.XY(i,s) = AB.X(i,s) * TDAN(i,s) + PDA.X(i,s)$$

$$AB.Y(i,s) = AB.X(i,s) + (DT) * \left(TRA.XY(i,s) + TCA.XY(i,s) - TDA.XY(i,s) \right)$$

$$ABD.Y(i,s) = AB.Y(i,s) * TADN(i,s)$$

Land use submodel

$$\# SLTT(j) = NSLTT(j)$$

$$\# SLA.X(j) = NSLA(j)$$

$$\# SLI.X(j) = NSLI(j)$$

$$\# SLT.X(j) = NSLT(j)$$

$$\# R.X\ (j) = RN(j)$$

$$TIJ(j,i) = \sum_v T(v,j,i) * RM(v)$$

$$\text{OSAB.XY}(j) = \sum_i \text{SLA.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right) / \sum_i \sum_i \text{SLA.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right)$$

$$\text{OSIN.XY}(j) = \sum_i \text{SLI.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right) / \sum_i \sum_i \text{SLI.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right)$$

$$\text{OSTN.XY}(j) = \sum_i \text{SLT.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right) / \sum_i \sum_i \text{SLT.X}(i) * \text{EXP}\left(-\text{TETA} * \text{TIJ}(j,i)\right)$$

$$\text{S1.Y}(j) = \text{SLA.X}(j) + (\text{DT})\left[\text{OSA}(j) * \sum_s \left(\text{TDA.XY}(j,s) - \text{TCA.XY}(j,s)\right)\right] + \text{SLAN.X}(j)$$

$$\text{S2.Y}(j) = \text{SLI.X}(j) + (\text{DT})\left[\text{OSI}(j) * \sum_t \left(\text{TRI.XY}(j,t) + \text{TDI.XY}(j,t) - \text{TCI.XY}(j,t)\right)\right] + \text{SLIN.X}(j)$$

$$\text{S3.Y}(j) = \text{SLT.X}(j) + (\text{DT})\left[\text{OST}(j) * \sum_l \left(\text{TRT.XY}(j,l) - \text{TCT.XY}(j,l)\right)\right] + \text{SLTN.X}(j)$$

$$\text{S4.Y}(j) = \text{R.X}(j) + \text{RNP.X}(j)$$

$$\text{SS.Y}(j) = \text{S1.Y}(j) + \text{S2.Y}(j) + \text{S3.Y}(j) + \text{S4.Y}(j)$$

$$\text{SLA.Y}(j) = \text{SLTT}(j)\left[\text{S1.Y}(j) / \text{SS.Y}(j)\right]$$

$$\text{SLI.Y}(j) = \text{SLTT}(j)\left[\text{S2.Y}(j) / \text{SS.Y}(j)\right]$$

$$\text{SLT.Y}(j) = \text{SLTT}(j)\left[\text{S3.Y}(j) / \text{SS.Y}(j)\right]$$

$$\text{R.Y}(j) = \text{SLTT}(j)\left[\text{S4.Y}(j) / \text{SS.Y}(j)\right]$$

Residential locatian submodel

$$\#\ \text{POTO.X}(f) = \text{POTON}(f)$$

$$\#\ \text{POTD.X}(g) = \text{POTDN}(g)$$

$$\#\ \text{DPOTO.X}(i,j,s,f) = \text{DPOTON}(j,j,s,f)$$

$$\#\ \text{DPOTD.X}(i,j,s,g) = \text{DPOTDN}(i,j,s,g)$$

$$\text{POTI.X} = \sum_f \text{POTO.X}(f) + \sum_g \text{POTD.X}(g)$$

$$\text{DPOTI.X}(j) = \sum_{i,s} \left(\sum_f \text{DPOTO.X}(i,j,s,f,) + \sum_g \text{DPOTD}(i,j,s,g)\right)$$

$$\text{TAS.X} = \text{P.X} / \text{POTI.X}$$

$$\text{O.X}(j,f) = \sum_t \left(\text{IND.X}(j,t) * \text{PROBI}(t,f)\right) + \sum_l \left(\text{TERZ.X}(j,l) * \text{PROBT}(l,f)\right)$$

$$Q.X(j,f) = \sum_i \sum_s DPOTO(i,j,s,f)$$

$$KO.X(j,f) = Q.X\,(j,f) \,/\, O.X(j,f)$$

$$O.Y\,(j,f) = \sum_t \Big(IND.Y(j,t) * PROBI(t,f) \Big) + \sum_l \Big(TERZ.Y(j,l) * PROBT(l,f) \Big)$$

$$Q.Y(j,f) = O.Y(j,f) * KO.X(j,f)$$

$$POTO.Y(f) = \sum_j Q.Y(j,f)$$

$$POTD.Y(g) = \Big[(P.Y/TAS.X) - \sum_f POTO.Y(f) \Big] * POTD.X(g) / \sum_g POTD.X(g)$$

$$A.Y(i,f) = \sum_v \sum_j \Big[Q.Y(j,f) * TPROB(f,v) * EXP\Big(-BETA(f) * TTJI(i,j,v)\Big) \Big] /$$

$$/ \sum_v \sum_i \sum_j \sum_f \Big[Q.Y(j,f) * TPROB(f,v) * EXP\Big(-BETA(f) * TTJI(i,j,v)\Big) \Big]$$

$$AT.Y(i,f) = \Big[A.Y(i,f) - A.Y_{min}(i,f) \Big] / \Big[A.Y_{max}(i,f) - A.Y_{min}(i,f) \Big]$$

$$RES.Y(i,s) = AB.Y(i,s) / \sum_i \sum_s AB.Y(i,s)$$

$$L.Y(i) = SLA.Y(i) / \sum_i SLA.Y(i)$$

$$U.Y(i,s,f) = \Big[K(i,f) * AT.Y(i,f) \Big] + \Big[H(i,s,f) * RES(i,s) \Big] + \Big[N(i,f) * L.Y(i) \Big]$$

$$B.Y(j,f) = \Big(\sum_i \sum_s \sum_v \Big[TPROB(f,v) * \Big(-BETA(f) * TTJI(i,j,v)\Big) \Big] * EXP\Big\{ -CSI(f) * \Big[\overline{U}(f) - U.Y(i,s,f) \Big] \Big\} \Big)^{-1}$$

$$DPOTO.Y(i,j,s,f,) = \sum_v \Big(Q.Y(i,f) * TPROB(f,v) * EXP\Big[-BETA(f) * TTJI(i,j,v)\Big] \Big) * EXP\Big\{ -CSI(f) \Big[\overline{U}(f) - U.Y(i,s,f) \Big] \Big\} * B.Y(j,f)$$

$$DPOTD.Y(i,j,s,g) = POTD.Y(g) * \Big[DPOTD.X(i,j,s,g) / POTD.X(g) \Big]$$

3. Fuzzy approach to the utility function

3.1. Utility function

It can be assumed that each point - in a set k of points of the space -, characterized by a set j of factors, has a given value, attributed to it by a set of consumers I, according to a utility function whose most general expression is

$$U_k^i = f\left[m_k(j)\ ,\gamma_j(i)\,;\ j \in J \right], \qquad (6)$$

where:

k is a destination point in the space for the consumer (k=1,..,K);

j is one of the factors characterizing point k (j=1,...,J);

i is the consumer (i=1,...,I);

U_k^i is the utility derived for consumer i by the choice of destination k;

$m_k(j)$ is the attraction of k according to the factor j, that is, the relative mass of k for the factor j;

$\gamma_j(i)$ is the weight of the factor j for the consumer i. Such weight is a function having a value in the range $[0,1]$ depending on the importance that factor j has for the consumer i $[\forall j \in J,\ 0 \leq \gamma_j(i) \leq 1]$.

The utility function (6) is a fuzzy utility function.

Firstly, the set of destination points of consumers form an ill-defined space, as consumers do not possess precise knowledge of the characteristics of the different points.

Secondly, the utility derived for a consumer i from the choice of a destination point k is a multidimensional and subjective idea: it is multidimensional because the consumer's journey to a point k is motivated by a set j of factors; it is subjective because, for each factor j, the consumer expresses a subjective assessment $\gamma_j(i)$ (Fustier, 1978).

3.2. The fuzzy utility function

With reference to section 3.1., the set of residential locations form from the families' viewpoint, an ill-defined residential space. Hence it can be assumed that for a type F family, each residential location would be characterized by the following set of locational factors:

M^f_{is} money allocated to nonlocational expenditures by a type f family, living in a housing type s in the residential zone i;

H_{is} number of units of housing type s in the zone i ;

A^f_i relative accessibility of the residential zone i for a type f family;

$N_{i(l)}$ set of environmental characteristics, l = 1, ..., L, of the residential zone i.

The utility derived for a type f family, in a residential location i, is, on the basis of (6) :

$$U^f_{is} = f\left[M^f_{is}, H_{is}, A^f_i, N_{i(l)}; \ \gamma^M_{is}(f), \ \gamma^H_{is}(f), \ \gamma^A_i(f), \ \gamma^N_i(f)\right]. \tag{7}$$

Making (7) explicit and using an expression derived from a Cobb-Douglas formulation, (7) can be rewritten as :

$$\ln U^f_{is} = \gamma^M_{is}(f) \cdot m^f_{is} + \gamma^H_{is}(f) \cdot h_{is} + \gamma^A_i(f) \cdot a^f_i + \sum_{l=1}^{L} \gamma^N_{i(l)}(f) \cdot n_{i(l)}, \tag{8}$$

where:

$m^f_{is} = \dfrac{M^f_{is}}{\sum_i \sum_s M^f_{is}}$ is the relative weight of factor M^f_{is}, $m \in M = [0,1]$;

$h_{is} = \dfrac{H_{is}}{\sum_i \sum_s H_{is}}$ is the relative weight of factor H_{is}, $h \in H = [0,1]$;

$a^f_i = \dfrac{A^f_i}{\sum_i A^f_i}$ is the relative weight of factor A^f_i, $a \in A = [0,1]$;

$n_{i(l)} = \dfrac{N_{i(l)}}{\sum_i N_{i(l)}}$ is the relative weight of factor $N_{i(l)}$, $n \in N = [0,1]$;

and

$\gamma^M_{is}(f)$, $\gamma^H_{is}(f)$, $\gamma^A_i(f)$, $\gamma^N_{i(l)}(f)$ are functions which take value in the range $[0,1]$,

$\forall x \in X\ ,\ \ 0 \leq \gamma_x\,(f) \leq 1,$

where X is the set of factors.

These functions depend on the characteristics and preferences of the type family. Precisely, the values of these functions are the assessments expressed for each factor by type f family.

$\gamma^M_{is}(f),\ \gamma^H_{is}(f),\ \gamma^A_i(f),\ \gamma^N_{i(l)}(f)$ may be rewritten as follows:

$$\gamma^M_{is}(f) = \mu_f(m^f_{is})$$

$$\gamma^H_{is}(f) = \mu_f(h_{is})$$

$$\gamma^A_i(f) = \mu_f(a^f_i)$$

$$\gamma^N_{i(l)}(f) = \mu_f(n_{i(l)}),$$

where $\mu_f(m^f_{is}), \ldots, \mu_f(n_{i(l)})$ express the degree of membership of each factor to the utility functions for the f type family (i.e. they measure to what extent, for the family, each factor contributes to the utility level).

The (8) represents the real utility function for a f type family, working in j, living in a housing type s in residential zone i.

4. Structural stability

4.1. A simplified version of the model

The outcomes of complex models, such as the model described in section 2.2., are not easy to foresee and, at times, completely unexpected.

Two aspects of this counter-intuitive behaviour are:

a. inertiae, that we see when the outputs are substantially unvarying, even for large variations in some model parameters;

b. instabilities, that we see when the outputs change considerably for small variations in the parameters.

When unexpected inertiae and instabilities are checked in the modelled system, this contributes significatively to the validation of the model.

Therefore, the analysis of the sensitivity of outputs vs parameters is a fundamental step, when dealing with complex models.

Yet, these models often have a very large number of parameters, to the point that it is difficult, if not impossible (because of the number of the calculations involved) to carry out a systematic analysis with respect to the full set of parameters.

It follows, then, that it is useful to try to know a priori, in some way, the subset of parameters by which the behaviour of the model might be more critical.

To this end, in the present section, we will develop a simplified version of the model described in 2.2., examining its properties - analytically - to identify a condition of instability in connection to which it becomes interesting - by simulation - to explore the behaviour of the original model.

The simplification has two basic aspects:

a. we focus on the core of the model, that is, on the housing market, and, in particular, on the levels of residential mobility associated with the market;

b. we eliminate the spatial and socioeconomic disaggregation.

On this basis, denoting by:

x the number of families which, at a given time, change dwelling;

y the number of dwellings that, at that time, are available for moving families,

the original model can be reduced to the following dynamic model:

$$\dot{\underline{\underline{x}}} = \underline{\underline{f(x, c)}} , \tag{9}$$

where:

$\underline{\underline{x}} = (x,y)$ is the state vector;

$\underline{\underline{c}}$ is the parameter vector;

$\underline{\underline{f}}$ is the function which defines the change in state.

It must be pointed out that x also is the residential mobility level at a given time. With the same degree of approximation as in the original model, let us now assume the dynamic model to be linear; then (9) takes the form:

$$\dot{\underline{\underline{x}}} = \underline{\underline{A}}\,\underline{\underline{x}} + \underline{\underline{b}}\,, \tag{10}$$

where the elements of matrix A and vector b are the six elements of the parameters vector c previously mentioned.

It is possible to establish the sign of some of the parameters, making the following assumption (that must hold, as observation suggests, both for the simplified and the original models):

a. as x grows, coeteris paribus, $\dot{x}$ decreases and $\dot{y}$ increases (in other words, the larger is the number of families that move, the fewer are the "new" families that move and the greater is the offer of dwellings);

b. as y grows, coeteris paribus, $\dot{y}$ decreases and $\dot{x}$ increases (in other words, the larger is the number of dwellings available to families that move, the fewer are the "new" dwellings available and the greater is the demand for dwellings).

Thus, the variation of each of the two state variables is positively correlated to the level of the other variable and negatively correlated to the level of the variable itself, leading to the special form of (10) given by:

$$\begin{aligned} \dot{x} &= ay - mx + g \\ \dot{y} &= bx - nt + h\,, \end{aligned} \tag{11}$$

where a, b, m and n are positive but g and h may take positive or negative values.

The dynamic process described by (11) is known as the Richardson process (Richardson, 1960) (after the name of the author who used it to describe the armament of disarmament escalation involving two nations).

In sector 4.2. we will examine the most characteristic properties of such a process, interpreting them in terms of the housing market.

4.2. Qualitative analysis of Richardson process

Let us begin by considering the existence of equilibrium points in the dynamic process, setting $\dot{x} = 0$ and $\dot{y} = 0$.

We get:

$$\begin{aligned} ay - mx + g &= 0 \\ bx - ny + h &= 0, \end{aligned} \qquad (12)$$

these being the equations of two straight lines L_1 and L_2 in the phase plane:

$$\begin{aligned} L_1 &: y = (m/a)x - (g/a) \\ L_2 &: y = (b/n)x + (h/n) \, . \end{aligned} \qquad (13)$$

We have an equilibrium point when L_1 and L_2 intersect in the first quadrant (in fact, the equilibrium point coordinates (x_o, y_o) both satisfy (12), and x_o and y_o are, as they should be: $x_o > 0$ and $y_o > 0$).

Observing that both the slopes of L_1 and L_2 are positive, if:

- the slope of L_1 is steeper than that of L_2 :

$$(m/a) > (b/n) \quad \text{that is} \quad mn - ab > 0 \, , \qquad (14)$$

we have an equilibrium solution when:

$$ng + ah > 0 \quad \text{and} \quad bg + mh > 0 \, ; \qquad (15)$$

- the slope of L_1 is less steep than that of L_2 :

$$(m/a) < (b/n) < \quad \text{that is} \quad mn - ab < 0 \, , \qquad (16)$$

we have an equilibrium solution when:

$$ng + ah < 0 \quad \text{and} \quad bg + mh < 0 \, . \qquad (17)$$

As we will see below, the equilibrium (14) - (15) is stable, the equilibrium (16) - (17) is unstable. A rigorous proof of this is found in Braun (1975).

Let us now consider the disequilibrium states in the process, analysing for each point in the phase plane, the direction of change of the system (defined by the sign of $\dot{x}$ and $\dot{y}$).

It is possible to distinguish four types of behaviour, depending on wether the parameters vector $\underline{\underline{c}}$:

A. satisfies (14) and satisfies (15);

B. satisfies (14) and does not satisfy (15);

C. satisfies (16) and satisfies (17);

D. satisfies (16) and does not satisfy (17).

The four cases are illustrated in fig.7. The arrows in the figure indicate the direction of change of the system.

We observe that in all cases except C, for each of the areas (in the phase plane) defined by L_1 and L_2, we have only one direction of change. In case C it does not happen, and the dashed line divides two alternative directions of change. In the theory of dynamic systems, this line is called a "separatrix".

From the directions of change, moreover, we see that in case A equilibrium is stable, while in case C is unstable.

Let us now interpret the four cases in terms of residential mobility, If the values of the parameters are such that we are:

- in case A, then the residential mobility level settles at an equilibrium value;
- in case B, then the residential mobility level is diminuishing;
- in case D, then the residential mobility level is increasing;
- in case C, then the residential mobility level - depending on the system's initial conditions - increase or decrease, or else, which is very unlikely, it settles at an equilibrium value from which it will very easily move away

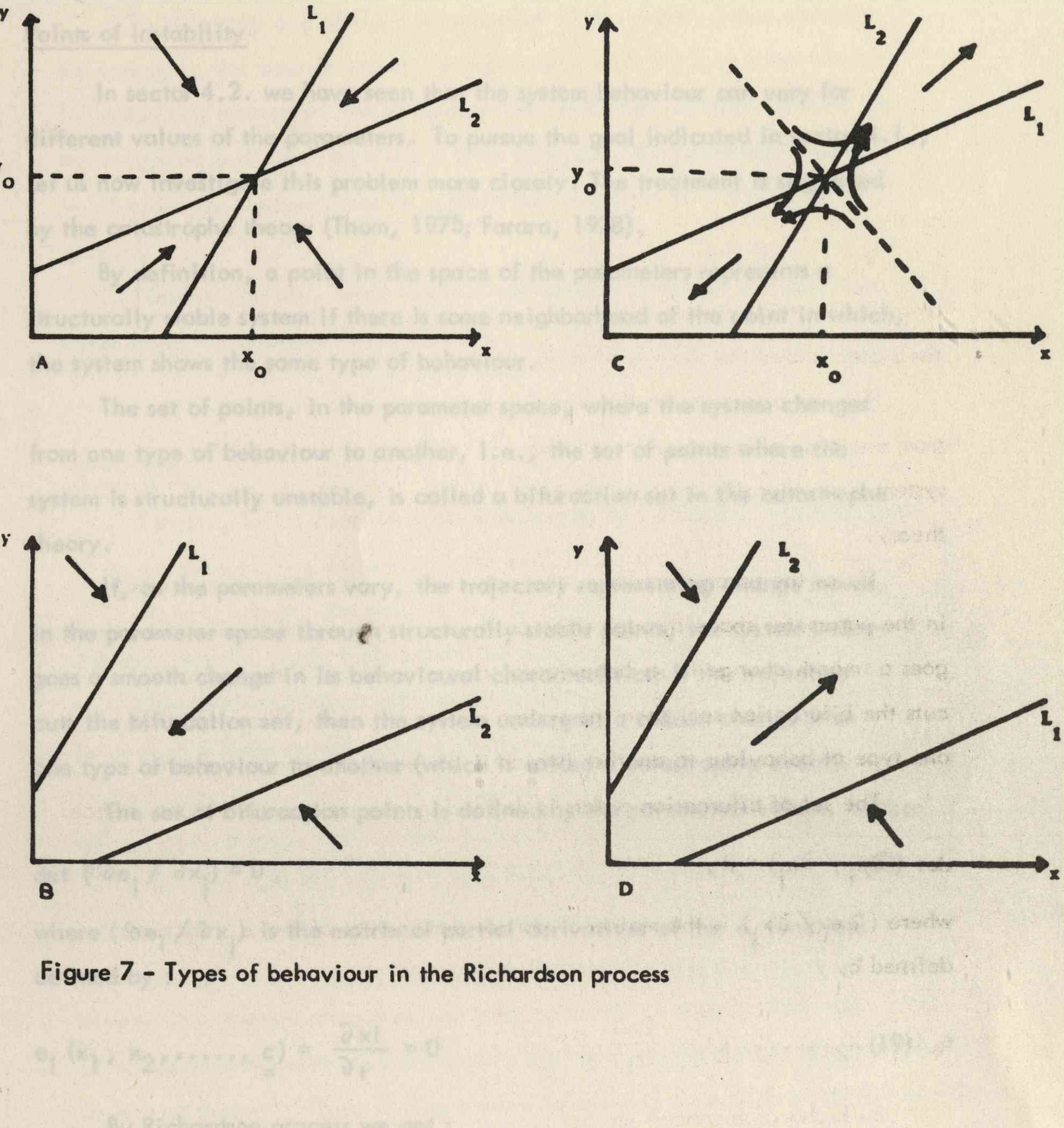


Figure 7 – Types of behaviour in the Richardson process

(even for small changes in the parameters) to fall back on one of the two foregoing cases.

4.3. Points of instability

In sector 4.2. we have seen that the system behaviour can vary for different values of the parameters. To pursue the goal indicated in sector 4.1., let us now investigate this problem more closely. The treatment is suggested by the catastrophe theory (Thom, 1975; Fararo, 1978).

By definition, a point in the space of the parameters represents a structurally stable system if there is some neighborhood of the point in which, the system shows the same type of behaviour.

The set of points, in the parameter space, where the system changes from one type of behaviour to another, i.e., the set of points where the system is structurally unstable, is called a bifurcation set in the catastrophe theory.

If, as the parameters vary, the trajectory representing change moves in the parameter space through structurally stable points, the system undergoes a smooth change in its behavioural characteristics; if the trajectory cuts the bifurcation set, then the system undergoes a sudden change from one type of behaviour to another (which is called a catastrophic event).

The set of bifurcation points is defined by the parameters values, hence:

$$\det \left(\partial e_i / \partial x_l \right) = 0 , \qquad (18)$$

where $(\partial e_i / \partial x_l)$ is the matrix of partial derivatives of the e_i functions defined by :

$$e_i \left(x_1 , x_2 , \ldots\ldots , \underline{\underline{c}} \right) = \frac{\partial xi}{\partial_t} = 0 \qquad (19)$$

By Richardson process we get :

$$\det \begin{pmatrix} -m & a \\ b & -n \end{pmatrix} = mn - ab = 0 . \qquad (20)$$

Consequently, the points in the space which satisfy the equation $mn - ab = 0$ are the bifurcation set in the system we are studying.

To illustrate one of the consequences of this result, let us consider, for example, the case in which, as parameters vary (*) they cross the bifurcation set; moreover, let us assume that, although parameters vary, we have $ng + ab > 0$ and $bg + mh > 0$; in this case, the residential mobility level suddenly changes from an equilibrium condition to a condition of growth.

Needless to say, other kinds of parameter variations will result in other types of changes.

5. Conclusion

Finally, let us consider a problem connected with the implementation of the model which is currently under way.

As it is known, the availability of computers makes it possible to apply complex models, study their behaviour and perfect the details so that in the end, simulated behaviour will adequately reproduce actual behaviour. It is evident that these models have a limited predictive capacity and, hence, limited usefulness. Such a risk is present in this case, too, considering that we are using a large scale model with relatively few observed data.

To eliminate this risk it is necessary that at the implementation stage we make sure that the number of independent parameters in the model be considerably lower than the number of observable variables. Under this condition, it is important to evaluate whether model disaggregation offers - in comparison to more elementary models having the same number of parameters - an actual benefit. Thus, it is necessary to develop a measure of the information gain associated with the model being considered. In this connection, the guidelines for further work are those suggested by Maciejowski (1978).

(*) Let us recall that, in the model described at section 2.2., the simplified submodel analyzed here is linked to other submodels, so that the values of the $\underline{\underline{c}}$ parameters are defined by the other submodels and, hence, they can vary in time.

To illustrate one of the consequences of this result, let us consider, for example, the case in which, as parameters vary (*) they cross the bifurcation set; moreover, let us assume that, although parameters vary, we have ng + ab > 0 and bg + mh > 0; in this case, the residential mobility level suddenly changes from an equilibrium condition to a condition of growth.

Needless to say, other kinds of parameter variations will result in other types of changes.

5. Conclusion

Finally, let us consider a problem connected with the implementation of the model which is currently under way.

As it is known, the availability of computers makes it possible to apply complex models, study their behaviour and perfect the details so that in the end, simulated behaviour will adequately reproduce actual behaviour. It is evident that these models have a limited predictive capacity and, hence, limited usefulness. Such a risk is present in this case, too, considering that we are using a large scale model with relatively few observed data.

To eliminate this risk it is necessary that at the implementation stage we make sure that the number of independent parameters in the model be considerably lower than the number of observable variables. Under this condition, it is important to evaluate whether model disaggregation offers - in comparison to more elementary models having the same number of parameters - an actual benefit. Thus, it is necessary to develop a measure of the information gain associated with the model being considered. In this connection, the guidelines for further work are those suggested by Maciejowski (1978).

(*) Let us recall that, in the model described at section 2.2., the simplified submodel analyzed here is linked to other submodels, so that the values of the c parameters are defined by the other submodels and, hence, they can vary in time.

References

Alonso W. (1964) Location and land use, Harward University Press, Cambridge, Mass..

Anas A. (1973) A dynamic disequilibrium model of residential location, Environment and Planning, 5, 633-647.

Bertuglia C.S., Occelli S., Rabino G.A., Salomone C., Tadei R. (1981) Un modello urbano a larga scala per l'area metropolitana di Torino, Working Paper, 1, IRES, Torino.

Bertuglia C.S., Cccelli S., Rabino G., Tadei R. (1980) A model of urban structure and development of Turin: theoretical aspects, Sistemi Urbani, 2, 59-90.

Braun M. (1975) Differential equations and their applications, Springer Verlag, New York.

Fararo T. J. (1978) An introduction to catastrophes, Behavioural Science, 23, 291-317.

Forrester J.W. (1969) Urban dynamics, M.I.T. Press, Cambridge, Mass..

Fustier B. (1978) Contribution à l'analyse spatiale de l'attraction imprécise, Document de Travail, 30, I.M.E., Université de Dijon, Dijon.

Herbert J.T., Stevens B.H. (1960) A model for the distribution of residential activity in urban areas, Journal of Regional Science, 2, 21-36.

IRES (1978) Metodologia per la formazione dei piani comprensoriali di trasporto (mimeo).

Lowry I.S. (1964) A model of a metropolis, Rand Corporation, Santa Monica.

Maciejowski J.M. (1978) The modelling of systems with small observation sets, Lecture Notes in Control and Information Sciences, 10, Springer Verlag, New York.

Ponsard C. (1978) On the imprecision of consumers' spatial preferences, Document de Travail, 26, I.M.E., Université de Dijon, Dijon.

Richardson L.F. (1960) Arms and insecurity, Boxwood Press, Pittsburgh.

Thom R. (1975) Structural stability and morphogenesis, Benjamin Press, Reading, Mass..

Wilson A. G. (1970) Entropy in urban and regional modelling, Pion, London.

Wilson A. G. (1974) Urban and regional models in geography and planning, Wiley and Sons, London.

ires ISTITUTO RICERCHE ECONOMICO - SOCIALI DEL PIEMONTE
VIA BOGINO 21 10123 TORINO