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IDENTIFICATION AND ESTIMATION OF TREATMENT EFFECTS IN THE PRESENCE OF NEIGHBOURHOOD INTERACTIONS

Giovanni Cerulli

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Identification and Estimation of Treatment Effects in the Presence of Neighbourhood Interactions

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ABSTRACT: This paper presents a parametric counter-factual model identifying Average Treatment Effects (ATEs) by Conditional Mean Independence when externality (or neighbourhood) effects are incorporated within the traditional Rubin's potential outcome model. As such, it tries to generalize the usual control-function regression, widely used in program evaluation and epidemiology, when SUTVA (i.e. Stable Unit Treatment Value Assumption) is relaxed. As by-product, the paper presents also `ntreatreg`, an author-written Stata routine for estimating ATEs when social interaction may be present. Finally, an instructional application of the model and of its Stata implementation through two examples (the first on the effect of housing location on crime; the second on the effect of education on fertility), are showed and results compared with a no-interaction setting.

Keywords: ATEs, Rubin's causal model, SUTVA, neighbourhood effects, Stata command.

JEL Codes: C21, C31, C87

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1. INTRODUCTION

In observational program evaluation studies, aimed at estimating the effect of an intervention on the outcome of a set of targeted individuals, it is generally assumed that *"the treatment received by one unit does not affect other units' outcome"* (Cox, 1958). Along with other fundamental assumptions - such as, for instance, the conditional independence assumption, the exclusion restriction provided by instrumental-variables estimation, or the existence of a forcing-variable in regression discontinuity design - the no-interference assumption is required in order to obtain a consistent estimation of the (average) treatment effects (ATEs). It means that, if interference (or interaction) among units is assumed, traditional program evaluation methods such as control-function regression, selection models, matching or reweighting are bound to be biased estimations of the actual treatment effect.

Rubin (1978) calls this important assumption as Stable-Unit-Treatment-Value-Assumption (SUTVA), whereas Manski (2013) refers to Individualistic-Treatment-Response (ITR) to emphasize that this poses a restriction in the form of the treatment response function that the analyst considers. SUTVA (or ITR) implies that the treatment applied to a specific individual affects only the outcome of that individual, so that potential "externality effects" flowing for instance from treated to untreated subjects are sharply ruled out.

In this paper, we aim at removing this hypothesis to understand what happens to the estimation of the effect of a binary policy (treatment) in the presence of neighbourhood (externality) effects taking place among supported (treated) and non-supported (untreated) units.

Epidemiological studies have addressed this hot topic although restricting the analysis to experimental settings where treatment randomization is assumed (see, for instance: Rosenbaum, 2007; Hudgens and Halloran, 2008; Tchetgen-Tchetgen and VanderWeele, 2010; Robins et al., 2000). Differently, this paper moves along the line traced by econometric studies normally dealing with non-experimental settings where sample selection is the rule (i.e., no random draw is assumed) and an ex-post evaluation is thus envisaged (Sobel, 2006). In particular, we work within the binary potential outcome model that in many regards we aim at generalizing for taking into account neighbourhood effects. Our theoretical reference may be found in some previous works dealing with treatment effect identification in the presence of externalities and in particular in the papers by Manski (1993; 2013).

Moreover, as by-product, this work also presents a Stata routine, `ntreatreg`, for estimating Average Treatments Effects (ATEs) when neighbourhood effects are taken into account.

The paper is organized as follows: section 2 presents some related literature and positions our approach within the Manski's notion of "endogenous" neighbourhood effects; section 3 sets out the model, its assumptions and propositions; section 4 presents the model's estimation procedure; section 5 puts forward the Stata implementation of the model via the author-written routine `ntreatreg`, and then provides two applications: one on the effect of housing location on crime; and one on the effect of education on fertility; section 6, finally, concludes the paper.

2. RELATED LITERATURE

The literature on the estimation of treatment effects under potential interference among units is a recent and challenging field of statistical and econometric study. So far, however, only few papers have dealt formally with this relevant topic.

Rosenbaum (2007) was among the first scholars paving the way to generalize the standard randomization statistical approach for comparing different treatments to the case of units' interference. He presented a statistical model in which unit's response depends not only on the treatment individually received, but also on the treatment received by other units', thus showing how it is possible to test the null-hypothesis of no interference in a random assignment setting where randomization occurs within pre-specified groups and interference between groups is ruled out.

On the same vein, Sobel (2006) provided a definition, identification and estimation strategy for traditional average treatment effect estimators when interference between units is allowed, by taking as example the "Moving To Opportunity" (MTO) randomized social experiment. In his paper, he uses interchangeably the term interference and spillover to account for the presence of such a kind of externality. Interestingly, he shows that a potential *bias* can arises when no-interference is erroneously assumed, and defines a series of direct and indirect treatment effects that may be identified under reasonable assumptions. Moreover, this author shows some interesting links between the form of his estimators under interference and the Local Average Treatment Effect (LATE) estimator provided by Imbens and Angrist (1994), thus showing that – under interference – treatment effects can be identified only on specific sub-populations.

The paper by Hudgens and Halloran (2008) is probably the most relevant of this literature, as these authors develop a rather general and rigorous modelling of the statistical treatment setting under randomization when interference is potentially present. Furthermore, their approach paves the way also for extensions to observational settings. Starting from the same two-stage randomization approach of Rosenbaum (2007), these authors manage to go substantially farther by providing a precise characterization of the causal effects with interference in randomized trials encompassing also the Sobel's approach. They define *direct*, *indirect*, *total* and *overall* causal effects showing the relation between these measures and providing an unbiased estimator of the upper bound of their variance.

Tchetgen-Tchetgen and VanderWeele (2010)'s paper follows in the footsteps traced by the approach of Hudgens and Halloran (2008) and provides a formal framework for statistical inference on population average causal effects in a finite sample setting with interference when the outcome variable is binary. Interestingly, they also present an original inferential approach for observational studies based on a generalization of the Inverse Probability Weighting (IPW) estimator when interference is present. Unfortunately, they do not provide the asymptotic variances for such estimators.

Aronow and Samii (2013) finally generalizes the approach proposed by Hudgens and Halloran (2008) going beyond the hierarchical experiment setting and providing a general variance estimation including covariates adjustment.

Previous literature assumes that the potential outcome y of unit i is a function of the treatment received by such a unit (w_i) and the treatment received by all the other units ($\mathbf{w}_{-i}$), that is:

$$y_i(w_i; \mathbf{w}_{-i}) \quad (1)$$

entailing that – with N units and a binary treatment for instance – a number of 2^N potential outcomes may arise. Nevertheless, an alternative way of modelling unit i's potential outcome may be that of assuming:

$$y_i(w_i; \mathbf{y}_{-i}) \quad (2)$$

where $\mathbf{y}_{-i}$ is the (N-1)x1 vector of other units' potential outcomes excluding unit i's potential outcome. The notion of interference entailed by expression (2) is different from that implied by expression (2). The latter, however, is well consistent with the notion of "endogenous" neighbourhood effects provided by Manski (1993, pp. 532-533). Manski, in fact, identifies three types of effects corresponding to three arguments of the individual (potential) outcome equation incorporating social effects[1]:

1. *Endogenous effects*. Such effects entail that the outcome of an individual depends on the outcomes of other individuals belonging to his neighbourhood.
2. *Exogenous (or contextual) effects*. These effects concern the possibility that the outcome of an individual is affected by the exogenous idiosyncratic characteristics of the individuals belonging to his neighbourhood.
3. *Correlated effects*. They are effects due to belonging to a specific group and thus sharing some institutional/normative condition (that one can loosely define as "environment").

Contextual and correlated effects are to be assumed as exogenous, as they clearly depend on pre-determined characteristics of the individuals in the neighbourhood (case 2) or of the neighbourhood itself (case 3). Endogenous effects are on the contrary of broader interest, as they are affected by the behaviour (measured as "outcome") of other individuals involved in the same neighbourhood. This means that endogenous effects both comprise direct and indirect effects linked to a given external intervention on individuals. The model proposed in this paper incorporates the presence of endogenous neighbourhood effects as defined by Manski within a traditional binary counterfactual model and provides both an identification and an estimation procedure for the Average Treatment Effects (ATEs) in this specific case.

How can we position this paper within the literature? Very concisely, previous literature assumes that: (i) unit potential outcome depends on own treatment and other units' treatment; (ii) assignment is randomized or conditionally unconfounded; (iii) treatment is multiple; (iii) potential outcomes have a non-parametric form. This paper, instead, assumes that: (i) unit potential outcome depends on own treatment and other units' potential

[1] The literature is not homogeneous in singling out a unique name of such effects: although context-dependent, authors interchangeably refer to neighbourhood, social, club, interference or interaction effects.

outcome; (ii) assignment is mean conditionally unconfounded; (iii) treatment is binary; (iv) potential outcomes have a parametric form.

As such, this paper suggests a simple but workable way to relax SUTVA, one that seems rather easy to implement in many socio-economic contexts of application.

3. A BINARY TREATMENT MODEL WITH "ENDOGENOUS" NEIGHBOURHOOD EFFECTS

This section presents a model for estimating the average treatment effects (ATEs) of a policy program (or a treatment) in a non-experimental setting in the presence of "endogenous" neighbourhood (or externality) interactions. We consider a *binary* treatment variable w - taking value 1 for treated and 0 for untreated units - assumed to affect an *outcome* (or *target*) variable y that can take a variety of forms.

Some notation can help in understanding the setting: N is the number of units involved in the experiment; N_1, the number of treated units; N_0 the number of untreated units; w_i the treatment variable assuming value "1" if unit i is treated and "0" if untreated; y_{1i} is the outcome of unit i when she is treated; y_{0i} is the outcome of unit i when she is untreated; $\mathbf{x}_i = (x_{1i}, x_{2i}, x_{3i}, \ldots, x_{Mi})$ is a row vector of M *exogenous* observable characteristics for unit $i = 1, \ldots, N$.

To begin with, as usual in this literature, we define the unit i's *Treatment Effect* (TE) as:

$$TE_i = y_{1i} - y_{0i} \qquad (3)$$

TE_i is equal to the difference between the value of the target variable when the individual is treated (y_1), and the value assumed by this variable when the *same* individual is untreated (y_0). Since TE_i refers to *the same individual at the same time*, the analyst can observe just one of the two quantities feeding into (3) but never both. For instance, it might be the case that we can observe the investment behaviour of a supported company, but we cannot know what the investment of this company would have been, had it not been supported, and vice versa. The analyst faces a fundamental *missing observation problem* (Holland, 1986) that needs to be tackled econometrically in order to recover reliably the causal effect via some specific imputation technique (Rubin, 1974; 1977).

Both y_{1i} and y_{0i} are assumed to be independent and identically distributed (i.i.d.) random variables, generally explained by a structural part depending on observable factors and a non-structural one depending on an unobservable (error) term. Nevertheless, recovering the entire distributions of y_{1i} and y_{0i} (and, consequently, the distribution of the TE_i) may be too demanding without very strong assumptions, so that the literature has focused on estimating specific moments of these distributions and in particular the "mean", thus defining the so-called population Average Treatment Effect (hereinafter ATE), and ATE *conditional on* $\mathbf{x}_i$ (i.e., ATE($\mathbf{x}_i$)) of a policy intervention as:

$$ATE = E(y_{i1} - y_{i0}) \qquad (4)$$

$$ATE(\mathbf{x}_i) = E(y_{i1} - y_{i0} \mid \mathbf{x}_i) \qquad (5)$$

where E(·) is the mean operator. ATE is equal to the difference between the average of the target variable when the individual is treated (y_1), and the average of the target variable when the same individual is untreated (y_0). Observe that, by the law of iterated expectations, ATE = $E_{\mathbf{x}}\{ATE(\mathbf{x})\}$.

Given the definition of the unconditional and conditional average treatment effect in (4) and (5) respectively, it is immediate to define the same parameters in the sub-population of treated (ATET) and untreated (ATENT) units, i.e.:

$$ATET = E(y_{i1}-y_{i0} \mid w_i=1)$$
$$ATET(\mathbf{x}_i) = E(y_{i1} - y_{i0} \mid \mathbf{x}_i, w_i=1)$$

and

$$ATENT = E(y_{i1}-y_{i0} \mid w_i=0)$$
$$ATENT(\mathbf{x}_i) = E(y_{i1} - y_{i0} \mid \mathbf{x}_i, w_i=0)$$

The aim of this paper is to provide consistent parametric estimation of all previous quantities (we refer to as ATEs) in the presence of neighbourhood effects.

To that end, we start with what is *observable* to the analyst in such a setting, i.e. the actual status of the unit *i*, that can be obtained as:

$$y_i = y_{0i} + w_i\,(y_{1i} - y_{0i}) \qquad (6)$$

Equation (6) is known as the Rubin's potential outcome model (POM), and it is the fundamental relation linking the unobservable with the observable outcome. Given Eq. (6), we first set out all the assumptions behind the next development of the proposed model.

Assumption 1. *Unconfoundedness* (or CMI). Given the set of random variables $\{y_{1i}, y_{1i}, w_i, \mathbf{x}_i\}$ as defined above, the following equalities hold:

$$E(y_{ig} \mid w_i, \mathbf{x}_i) = E(y_{ig} \mid \mathbf{x}_i) \quad \text{with } g = \{0,1\}$$

Hence, throughout this paper, we will assume unconfoundedness, i.e. *Conditional Mean Independence* (CMI) to hold. As we will show, CMI is a sufficient condition for identifying ATEs also when neighbourhood effects are considered.

Once CMI has been assumed, we then need to model the potential outcomes y_{0i} and y_{1i} in a proper way so to get a representation of the ATEs (i.e., ATE, ATET and ATENT) taking into account the presence of endogenous externality effects. In this paper, we simplify further our analysis by assuming some restrictions in the form of the potential outcomes.

Assumption 2. *Restrictions on the form of the potential outcomes*. Consider the general form of the potential outcome as expressed in (2), and assume this relation to depend parametrically on a vector of real numbers $\boldsymbol{\theta} = (\boldsymbol{\theta}_0; \boldsymbol{\theta}_1)$. We assume that:

$$y_{1i}(w_i; \mathbf{x}_i; \boldsymbol{\theta}_1)$$

and

$$y_{0i}(w_i; \mathbf{x}_i; \mathbf{y}_{1,-i}; \boldsymbol{\theta}_0)$$

Assumption 2 poses two important restrictions to the form given to the potential outcomes: (i) it makes them dependent on some unknown parameters $\boldsymbol{\theta}$ (i.e., parametric form); (ii) it entails that the externality effect occurs only in one direction, from the treated individuals to the untreated, while the other way round is ruled out.

Assumption 3. *Linearity and weighting-matrix*. We assume that the potential outcomes are linear in the parameters, and that a *N*x*N* weighting-matrix $\boldsymbol{\Omega}$ of exogenous constant numbers is known.

Under Assumptions 1, 2 and 3, the model takes on this form:

$$\begin{cases} y_{1i} = \mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i} \\ y_{0i} = \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma s_i + e_{0i} \\ s_i = \sum_{j=1}^{N_1} \omega_{ij} y_{1j} \\ y_i = y_{0i} + w(y_{1i} - y_{0i}) \\ \sum_{j=1}^{N_1} \omega_{ij} = 1 \\ i = 1, \ldots, N \\ j = 1, \ldots, N_1 \\ \text{CMI holds} \end{cases} \quad (7)$$

where μ_1 and μ_0 are scalars, $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_1$ are two unknown vector parameters defining the different response of unit i to the vector of covariates $\mathbf{x}$, e_0 and e_1 are two random errors with zero unconditional variance and s_i represents unit i-th neighbourhood effect due to the treatment administrated to units j (j = 1, ..., N_1). Observe that, by linearity, we have that:

$$s_i = \begin{cases} \sum_{j=1}^{N_1} \omega_{ij} y_{1j} & \text{if } i \in \{w=0\} \\ 0 & \text{if } i \in \{w=1\} \end{cases} \quad (8)$$

where the parameter ω_{ij} is the generic element of the weighting matrix $\boldsymbol{\Omega}$ expressing some form of *distance* between unit i and unit j. Although not strictly required for consistency, we also assume that these weights add to one, i.e.

$$\sum_{j=1}^{N_1} \omega_{ij} = 1 .$$

In short, previous assumptions say that units i neighbourhood effect takes the form of a weighted-mean of the outcomes of treated units and that this "social" effect has an impact only on unit i's outcome when this unit is untreated.

As a consequence, by substitution of (8) into (7), we get that:

$$y_{0i} = \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma \sum_{j=1}^{N_1} \omega_{ij} y_{1j} + e_{0i} \quad (9)$$

making clear that untreated unit's i outcome is a function of its own idiosyncratic characteristics ($\mathbf{x}_i$), the weighted outcomes of treated units multiplied by a sensitivity parameter γ, and a standard error term.

We state now a series of propositions implied by previous assumptions.

Proposition 1. *Formula of* ATE *with neighbourhood interactions*. Given assumptions 2 and 3 and the implied equations established in (7), the average treatment effect (ATE) with neighbourhood interactions takes on this form:

$$\text{ATE} = \text{E}(y_{1i} - y_{0i}) = \mu + \bar{\mathbf{x}}_i\boldsymbol{\delta} - \left(\sum_{j=1}^{N_1} \omega_{ij} \bar{\mathbf{x}}_j \right) \gamma \boldsymbol{\beta}_1 \quad (10)$$

where $\bar{\mathbf{x}}_i = \text{E}(\mathbf{x}_i)$ is the unconditional mean of the vector $\mathbf{x}_i$, and $\mu = \mu_1 - \mu_0 - \gamma\mu_1$. The proof is in Appendix. See A1.

Indeed, by the definition of ATE as given in (4) and by (7), we can immediately show that for such a model:

$$\text{ATE} = \text{E}(y_{1i} - y_{0i}) = \text{E}\left[\left(\mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i}\right) - \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma \sum_{j=1}^{N_1} \omega_{ij} y_{1j} + e_{0i} \right) \right] \quad (11)$$

where:

$$\sum_{j=1}^{N_1}\omega_{ij}y_{1j}=\sum_{j=1}^{N_1}\omega_{ij}\left(\mu_1+\mathbf{x}_j\boldsymbol{\beta}_1+e_{1j}\right)=$$
$$\mu_1\sum_{j=1}^{N_1}\omega_{ij}+\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\boldsymbol{\beta}_1+\sum_{j=1}^{N_1}\omega_{ij}e_{1j}= \tag{12}$$
$$\mu_1+\left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\boldsymbol{\beta}_1+\sum_{j=1}^{N_1}\omega_{ij}e_{1j}$$

and by developing ATE further using Eq. (11), we finally get the result in (10).

Proposition 2. *Formula of* ATE($\mathbf{x}_i$) *with neighbourhood interactions*. Given assumptions 2 and 3 and the result in proposition 1, we have that:

$$\mathrm{ATE}(\mathbf{x}_i)=\mathrm{ATE}+(\mathbf{x}_i-\bar{\mathbf{x}})\boldsymbol{\delta}+\sum_{j=1}^{N_1}\omega_{ij}(\bar{\mathbf{x}}-\mathbf{x}_j)\gamma\boldsymbol{\beta}_1 \tag{13}$$

where it is now easy to see that ATE $=\mathrm{E}_{\mathbf{x}}\{\mathrm{ATE}(\mathbf{x})\}$. The proof is in Appendix. See A2.

Proposition 3. *Baseline random-coefficient regression*. By substitution of equations (7) into the POM of Eq. (6), we obtain the following random-coefficient regression model (Wooldridge, 1997):

$$y_i=\eta+w_i\cdot\mathrm{ATE}+\mathbf{x}_i\boldsymbol{\beta}_0+w_i(\mathbf{x}_i-\bar{\mathbf{x}})\boldsymbol{\delta}+w_i\sum_{j=1}^{N}\omega_{ij}w_j(\bar{\mathbf{x}}-\mathbf{x}_j)\gamma\boldsymbol{\beta}_1+e_i \tag{14}$$

where, $\eta=\mu_0+\gamma\mu_1$ $\quad\boldsymbol{\delta}=\boldsymbol{\beta}_1-\boldsymbol{\beta}_0$

and

$$e_i=\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j}+e_{0i}+w_i(e_{1i}-e_{0i})-w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j}$$

The proof is in Appendix. See A3.

Proposition 4. *Ordinary Least Squares (OLS) consistency*. Under assumption 1 (CMI), 2 and 3, the error tem of regression (14) has zero mean conditional on (w_i, $\mathbf{x}_i$), i.e.:

$$\mathrm{E}\left(e_i\,|w_i,\mathbf{x}_i\right)=\mathrm{E}\left(\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j}+e_{0i}+w_i(e_{1i}-e_{0i})-w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j}\,|w_i,\mathbf{x}_i\right)=0 \tag{15}$$

thus implying that Eq. (14) is a regression model whose parameters can be *consistently* estimated by Ordinary Least Squares (OLS). The proof is in Appendix. See A4.

Once a consistent estimation of the parameters of (14) is obtained, we can estimate ATE directly from the regression, and ATE($\mathbf{x}_i$) by plugging the estimated parameters into formula (11). This is because ATE($\mathbf{x}_i$) becomes a function of consistent estimates, and thus consistent itself:

$$\mathrm{plim}\ \mathrm{ATE}(\mathbf{x}_i)=\mathrm{ATE}(\mathbf{x}_i)$$

where $\mathrm{ATE}(\mathbf{x}_i)$ is the plug-in estimator of ATE($\mathbf{x}_i$). Observe, however, that the (exogenous) weighting matrix $\boldsymbol{\Omega}=[\omega_{ij}]$ needs to be provided in advance.

Once the formulas for ATE and ATE($\mathbf{x}_i$) are available, it is also possible to recover the Average Treatment Effect on Treated (ATET) and on non-Treated (ATENT) as:

$$\text{ATET} = \text{ATE} + \frac{1}{\sum_{i=1}^{N} w_i} \sum_{i=1}^{N} w_i \left[(\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + \sum_{j=1}^{N_1} \omega_{ij} (\bar{\mathbf{x}} - \mathbf{x}_j) \gamma \boldsymbol{\beta}_1 \right] \quad (16)$$

and:

$$\text{ATENT} = \text{ATE} + \frac{1}{\sum_{i=1}^{N} (1-w_i)} \sum_{i=1}^{N} (1-w_i) \left[(\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + \sum_{j=1}^{N_1} \omega_{ij} (\bar{\mathbf{x}} - \mathbf{x}_j) \gamma \boldsymbol{\beta}_1 \right] \quad (17)$$

These quantities are functions of observable components and parameters consistently estimated by OLS (see next section). Once these estimates are available, standard errors for ATET and ATENT can be correctly obtained via bootstrapping (see Wooldridge, 2010, pp. 911-919).

4. ESTIMATION

Starting from previous section's results, a simple protocol for estimating ATEs can be suggested. Given an i.i.d. sample of observed variables for each individual i:

$$\{y_i, w_i, \mathbf{x}_i\} \text{ with } i = 1, \ldots, N$$

1. provide a weighting matrix $\Omega=[\omega_{ij}]$ measuring some type of distance between the generic unit i (untreated) and unit j (treated);
2. estimate by an OLS a regression model of:

$$y_i \quad \text{on} \quad \left\{1,\ w_i,\ \mathbf{x}_i,\ w_i(\mathbf{x}_i - \bar{\mathbf{x}}),\ w_i \sum_{j=1}^{N} \omega_{ij} w_j (\bar{\mathbf{x}} - \mathbf{x}_j) \right\}$$

3. obtain $\left\{\hat{\boldsymbol{\beta}}_0,\ \hat{\boldsymbol{\delta}},\ \hat{\gamma},\ \hat{\boldsymbol{\beta}}_1\right\}$ and put them into the formulas of ATEs.

By comparing for instance the formula of ATE *with* ($\gamma \neq 0$) and *without* ($\gamma = 0$) neighbourhood effect, we get the *neighbourhood-bias* defined as:

$$\text{Bias} = \left| \text{ATE}_{\text{no-neigh}} - \text{ATE}_{\text{with-neigh}} = \left(\sum_{j=1}^{N_1} \omega_{ij} \bar{\mathbf{x}}_j \right) \gamma \boldsymbol{\beta}_1 \right| \quad (18)$$

This can also be seen as the *externality effect* produced by the evaluated policy: it depends on the weights employed, on the average of the observable confounders considered into **x**, and on the magnitude of the coefficients γ and $\boldsymbol{\beta}_1$.

Observe that such bias may be positive as well as negative. Furthermore, by defining:

$$\gamma \boldsymbol{\beta}_1 = \boldsymbol{\lambda} \quad (19)$$

it is also possible to test whether this bias is or is not statistically significant by simply testing the following null-hypothesis:

$$\text{H}_0: \ \lambda_1 = \lambda_2 = \ldots = \lambda_M = 0$$

If this hypothesis is rejected, we cannot exclude that neighbourhood effects are pervasive, thus affecting significantly the estimation of the causal parameters ATEs. Finally, in a similar way, we can also get an estimation of the neighbourhood-bias for ATET and ATENT.

5. STATA IMPLEMENTATION VIA `ntreatreg`

The previous model can be easily estimated by using the author-written Stata routine `ntreatreg`.

The syntax of `ntreatreg` is a very common one for a Stata command and takes on this form:

```
ntreatreg outcome treatment
varlist hetero(varlist_h)
spill(matrix) graphic
```

where:

`outcome`: is the *y* of the previous model, i.e. the target variable of the policy considered.

`treatment`: is the *w* of the previous model, i.e. the binary policy (treatment) indicator.

`varlist`: is the **x** of the previous model, i.e. the vector of observable unit characteristics.

`hetero(``varlist_h`**`)`**: is an optional subset of **x** to allow for observable heterogeneity.

`spill(``matrix`**`)`**: is the weighting-matrix **Ω**, to be provided by the user.

`graphic`: returns a graph of the distribution of ATE(**x**), ATET(**x**) ans ATENT(**x**).

In the next two sub-sections we provide two instructional applications of the model presented in this paper and of its Stata implementation: the first one on the effect of housing location on crime; the second one on the effect of education on fertility.

Results are also compared with a no-interaction setting.

5.1 Example 1: the effect of location on crime

As a first application, we consider the dataset "SPATIAL_COLUMBUS.DTA" provided by Anselin (1988) containing information on property crimes in 49 neighbourhoods in Columbus, Ohio (US), in 1980. A total of 22 variables forms this dataset. The aim of this instructional application is that of evaluating the impact of *housing location* on *crimes*, i.e. the causal effect of the variable "cp" - taking value 1 if the neighbourhood is located in the "core" of the city and 0 if located in the "periphery" - on the number of residential burglaries and vehicle thefts per thousand households (i.e., the variable "crime"). Several conditioning (or confounding) observable factors are included in the dataset, but here we only consider two main factors, that is, the household income in $1,000 ("inc") and the housing value in $1,000 ("hoval"). We are interested in detecting the effect of housing location on the number of crimes in such a setting, by taking into account possible *interactions* among neighbourhoods. More in detail, our conjecture is that: "the number of crimes occurring in a peripheral neighbourhood (that is an 'untreated' unit) is not only affected by the income and the value of houses located within its boundaries, but also by the number of crimes occurred in core-neighbourhoods (i.e., the 'treated' units)", by assuming that this effect is proportional to the "distance" – measured by geographical coordinates – between the peripheral neighbourhood and the set of core-neighbourhoods. In what follows, the estimation steps with Stata commands.

Step 0. INPUT DATA FOR THE REGRESSION MODEL

y: `crime`
w: `cp`
x: `inc hoval`
Matrix **Ω**: `W`

Step 1. LOAD THE STATA ROUTINE "NTREATREG" AND THE DATASET

```
. ssc install ntreatreg
. ssc install spatwmat // see package: sg162 from
http://www.stata.com/stb/stb60
. use "SPATIAL_COLUMBUS.DTA"
```

Step 2. PROVIDE THE MATRIX "OMEGA" (HERE WE CALL IT "W")

```
. spatwmat, name(W) xcoord($xcoord) ycoord($ycoord) band(0 $band) ///
standardize eigenval(E)   // this generates the inverse distance matrix W
```

```
The following matrices have been created:
1. Inverse distance weights matrix W (row-standardized)
   Dimension: 49x49
   Distance band: 0 < d <= 10
   Friction parameter: 1
   Minimum distance: 0.7
   1st quartile distance: 6.0
   Median distance: 9.5
   3rd quartile distance: 13.6
   Maximum distance: 27.0
   Largest minimum distance: 3.37
   Smallest maximum distance: 14.51
2. Eigenvalues matrix E
   Dimension: 49x1
```

Step 3. ESTIMATE THE MODEL USING "NTREATREG" TO GET THE "ATE" WITH NEIGHBORHOOD-INTERACTIONS

```
. set more off
. xi: ntreatreg crime cp inc hoval , hetero(inc hoval) spill(W) graphic
```

```
      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  7,    41) =   15.74
       Model |  9793.37437     7  1399.05348           Prob > F      =  0.0000
    Residual |  3644.84518    41  88.8986629           R-squared     =  0.7288
-------------+------------------------------           Adj R-squared =  0.6825
       Total |  13438.2195    48  279.962907           Root MSE      =  9.4286
------------------------------------------------------------------------------
       crime |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          cp |   9.492458   4.816401     1.97   0.056    -.2344611    19.21938
         inc |  -.4968051   .3653732    -1.36   0.181    -1.234691     .241081
       hoval |  -.2133293    .101395    -2.10   0.042    -.4181006    -.008558
     _ws_inc |   -1.19053   .9911119    -1.20   0.237    -3.192121    .8110612
   _ws_hoval |   .1440651   .2268815     0.63   0.529    -.3141313    .6022616
   z_ws_inc1 |  -5.719737   2.934276    -1.95   0.058    -11.64563    .2061538
 z_ws_hoval1 |   .3889889   .9016162     0.43   0.668    -1.431862     2.20984
       _cons |   34.78312   8.655264     4.02   0.000     17.30346    52.26279
-------------+----------------------------------------------------------------
```

```
. scalar ate_neigh = _b[cp]       // put ATE into a scalar
. rename ATE_x _ATE_x_spill       // rename ATE_x as _ATE_x_spill
. rename ATET_x _ATET_x_spill
. rename ATENT_x _ATENT_x_spill
```

Step 4. DO A TEST TO SEE IF THE COEFFICIENTS OF THE NEIGHBOURHOOD-EFFECT ARE JOINTLY ZERO

4.1. if one accepts the null Ho: $\gamma\boldsymbol{\beta}_0 = 0$ => the neighbourhood-effect is negligible;
4.2. if one does not accept the null => the neighbourhood-effect effect is relevant.

```
. test  z_ws_inc1 = z_ws_hoval1 = 0
 ( 1)  z_ws_inc1 - z_ws_hoval1 = 0
 ( 2)  z_ws_inc1 = 0

       F(  2,    41) =    2.35
            Prob > F =    0.1078  //  externality effect seems not significant
```

Step 5. ESTIMATE THE MODEL USING "IVTREATREG" (TO GET ATE "WITHOUT" NEIGHBOURHOOD-INTERACTIONS)

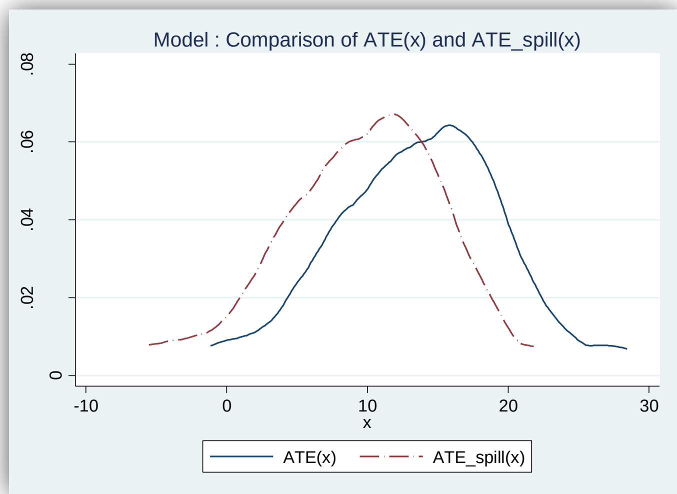
```
. xi: ivtreatreg crime cp inc hoval , hetero(inc hoval) model(cf-ols) graphic

      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  5,    43) =   19.84
       Model |  9375.05895     5  1875.01179           Prob > F      =  0.0000
    Residual |   4063.1606    43  94.4921069           R-squared     =  0.6976
-------------+------------------------------           Adj R-squared =  0.6625
       Total |  13438.2195    48  279.962907           Root MSE      =  9.7207

------------------------------------------------------------------------------
       crime |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          cp |   13.59008   4.119155     3.30   0.002     5.283016    21.89715
         inc |  -.8335211   .3384488    -2.46   0.018    -1.516068   -.1509741
       hoval |  -.1885477   .1036879    -1.82   0.076    -.3976543    .0205588
      _ws_inc |   -1.26008   1.004873    -1.25   0.217    -3.286599    .7664396
    _ws_hoval |   .2021829   .2300834     0.88   0.384    -.2618246    .6661904
       _cons |   46.52524   6.948544     6.70   0.000     32.51217    60.53832
------------------------------------------------------------------------------

. scalar ate_no_neigh = _b[educ7] // put ATE into a scalar
. di ate_no_neigh
```

Step 6. SEE THE MAGNITUDE OF THE NEIGHBORHOOD-INTERACTIONS BIAS

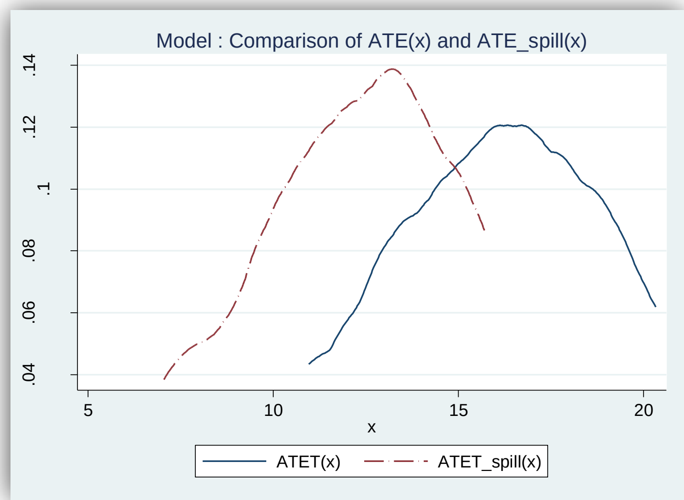
```
. scalar bias= ate_no_neigh - ate_neigh  // in level
. di bias
4.09  // the difference in level is around four crimes
. scalar bias_perc=(bias/ate_no_neigh)*100  // in percentage
. di bias_perc
30.15 // there is a 30% of bias due to neighbourhood interaction
```

Step 7. COMPARE GRAPHICALLY THE DISTRIBUTION OF ATE(**x**), ATET(**x**) and ATENT(**x**) WITH AND WITHOUT NEIGHBOURHOOD-INTERACTION

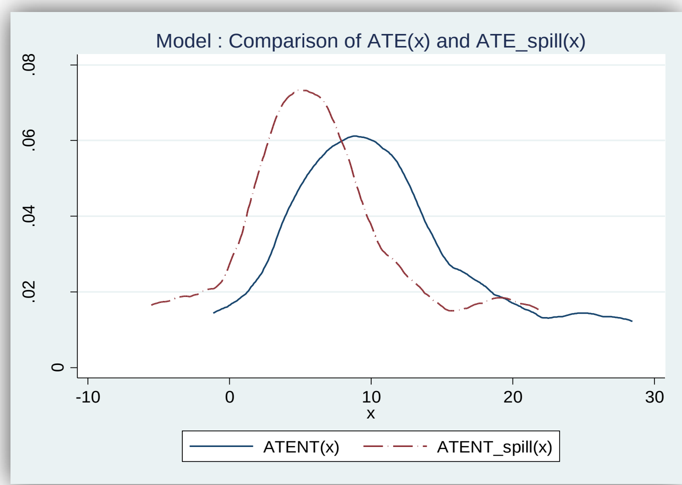
```
* ATE
twoway kdensity ATE_x , ///
|| ///
kdensity _ATE_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATE(x)" 2 "ATE_spill(x)")) ///
title("Model `model': Comparison of ATE(x) and ATE_spill(x)",
size(medlarge))
```




```
* ATET
twoway kdensity ATET_x , ///
|| ///
kdensity _ATET_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATET(x)" 2 "ATET_spill(x)")) ///
title("Model `model': Comparison of ATE(x) and ATE_spill(x)",
size(medlarge))
```



```
* ATENT
twoway kdensity ATENT_x , ///
|| ///
kdensity _ATENT_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATENT(x)" 2 "ATENT_spill(x)")) ///
title("Model  `model':  Comparison  of  ATE(x)  and  ATE_spill(x)",
size(medlarge))
```



As a conclusion, we can state that if the analyst does not consider "neighbourhood effects" she will "over-estimate" the actual effect of housing location on crime of around a 30%. However, the test seems to show that the neighbourhood effect is not relevant, as the coefficients of the neighbourhood component of regression (14) are not jointly significant.

5.2 *Example 2: the effect of education on fertility*

As a second application, we consider the dataset "FERTIL2_200.DTA" accompanying the manual "*Introductory Econometrics: A Modern Approach*" by Wooldridge (2000), where we consider only *N*=200 (out of 4,361) randomly drawn women in childbearing age in Botswana. The aim of this application is that of evaluating the impact of *education* on *fertility*, i.e. the causal effect of the variable "educ7" - taking value 1 if a woman has more than or exactly seven years of education and 0 otherwise - on the number of family children (the variable "children"). Several conditioning (or confounding) observable factors are included in the dataset, such as the age of the woman ("age"), whether or not the family owns a TV ("tv"), whether or not the woman lives in a city ("urban"), and so forth. We are particularly interested in detecting the effect of education on fertility in such a setting, by taking into account possible *peer-interactions* among women. In particular, our research presumption is that: "in choosing their 'desired' number of children, less educated women (the untreated ones) are not only affected by their own (idiosyncratic) characteristics (the **x**), but also by the number of children chosen by more educated women". The conjecture behind this statement is that less educated women might want to be as like as possible to more educated ones as a way to avoid some form of *social stigma*.

Step 0. INPUT DATA FOR THE REGRESSION MODEL

y: children
w: educ7
x: age agesq evermarr electric tv
Matrix **Ω**: dist

Step 1. LOAD THE STATA ROUTINE "NTREATREG" AND THE DATASET

```
. ssc install ntreatreg
. use "FERTIL2_200.DTA"
```

Step 2. PROVIDE THE MATRIX "OMEGA" (HERE WE CALL IT "dist")

```
. matrix dissimilarity dist = age agesq urban electric tv , corr // we use
correlation weights
. matewmf dist dist_abs, f(abs) // take the absolute values of the OMEGA
```

Step 3. ESTIMATE THE MODEL USING "NTREATREG" TO GET THE “ATE” WITH NEIGHBORHOOD-INTERACTIONS

```
. set more off
. xi: ntreatreg children educ7 age agesq evermarr electric tv  , ///
hetero(age agesq evermarr) spill(dist_abs) graphic


      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F( 12,   187) =   17.62
       Model |  493.24433    12  41.1036942           Prob > F      =  0.0000
    Residual |  436.33567   187  2.33334583           R-squared     =  0.5306
-------------+------------------------------           Adj R-squared =  0.5005
       Total |     929.58   199  4.67125628           Root MSE      =  1.5275
------------------------------------------------------------------------------
      children |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+--------------------------------------------------------------
         educ7 |  -.3869939   .2405745    -1.61   0.109    -.8615826    .0875948
           age |   -.004031   1.109614    -0.00   0.997    -2.193002     2.18494
         agesq |  -.0037554   .0098361    -0.38   0.703    -.0231595    .0156486
      evermarr |   .7954806   .3436893     2.31   0.022      .117474    1.473487
      electric |  -1.173366   .5034456    -2.33   0.021    -2.166529   -.1802034
            tv |    .358726   .6334492     0.57   0.572    -.8908988    1.608351
       _ws_age |  -.1171632   .1797361    -0.65   0.515    -.4717342    .2374077
     _ws_agesq |   .0013009   .0029585     0.44   0.661    -.0045354    .0071372
  _ws_evermarr |   .0212155   .5385761     0.04   0.969     -1.04125    1.083681
     z_ws_age1 |   5041.887   8015.575     0.63   0.530    -10770.69    20854.46
   z_ws_agesq1 |  -151.9131   230.3377    -0.66   0.510    -606.3075    302.4812
z_ws_evermarr1 |   93992.24   130909.8     0.72   0.474    -164257.6    352242.1
         _cons |   14492.64   11104.22     1.31   0.193    -7412.988    36398.27
------------------------------------------------------------------------------

. scalar ate_neigh = _b[educ7]  // put ATE into a scalar
. di ate_neigh
-.3869939
. rename ATE_x _ATE_x_spill     // rename ATE_x as _ATE_x_spill
. rename ATET_x _ATET_x_spill
. rename ATENT_x _ATENT_x_spill
```

Step 4. DO A TEST TO SEE IF THE COEFFICIENTS OF THE NEIGHBOURHOOD-EFFECT ARE JOINTLY ZERO

4.1. if one accepts the null Ho: $\gamma\boldsymbol{\beta}_0 = 0$ => the neighbourhood-effect is negligible;
4.2. if one does not accept the null => the neighbourhood-effect effect is relevant.

```
. test  z_ws_age1 = z_ws_agesq1 = z_ws_evermarr1 = 0

 ( 1)  z_ws_age1 - z_ws_agesq1 = 0
 ( 2)  z_ws_age1 - z_ws_evermarr1 = 0
 ( 3)  z_ws_age1 = 0
       F(  3,   187) =    2.49
            Prob > F =    0.0619    // social interaction significant at 6%
```

Step 5. ESTIMATE THE MODEL USING "IVTREATREG" (TO GET ATE "WITHOUT" NEIGHBOURHOOD-INTERACTIONS)

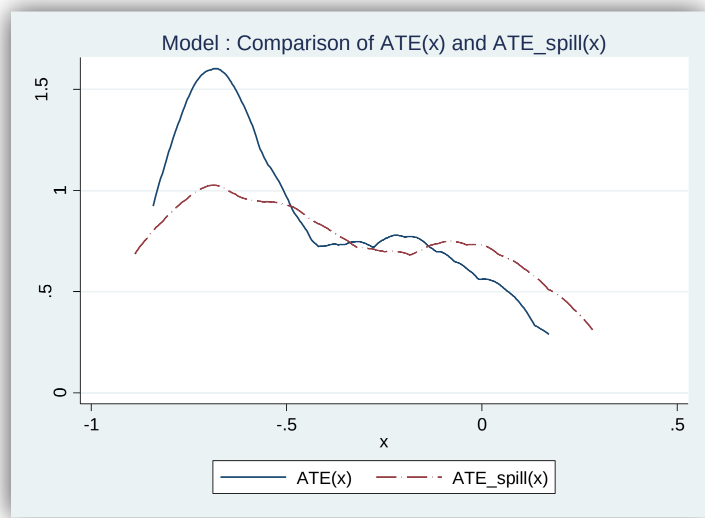
```
. xi: ivtreatreg children  educ7 age agesq evermarr electric tv  , ///
hetero(age agesq evermarr) model(cf-ols) graphic

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  9,   190) =   22.14
       Model |  475.829139     9  52.8699044           Prob > F      =  0.0000
    Residual |  453.750861   190  2.38816243           R-squared     =  0.5119
-------------+------------------------------           Adj R-squared =  0.4888
       Total |      929.58   199  4.67125628           Root MSE      =  1.5454

------------------------------------------------------------------------------
    children |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       educ7 |  -.4581193   .2417352    -1.90   0.060    -.9349488    .0187101
         age |   .4703103   .1252132     3.76   0.000     .2233237    .7172968
       agesq |  -.0053527   .0019811    -2.70   0.008    -.0092605    -.001445
    evermarr |   .7601864   .3439046     2.21   0.028     .0818249    1.438548
    electric |  -.8397923   .4060984    -2.07   0.040    -1.640833   -.0387517
          tv |   .1892151   .4754544     0.40   0.691    -.7486321    1.127062
     _ws_age |  -.1412403   .1788508    -0.79   0.431    -.4940286     .211548
   _ws_agesq |   .0018331   .0029337     0.62   0.533    -.0039537    .0076199
_ws_evermarr |   .0667193    .543741     0.12   0.902    -1.005825    1.139264
       _cons |  -6.409861    1.83986    -3.48   0.001    -10.03904   -2.780685
------------------------------------------------------------------------------

. scalar ate_no_neigh = _b[educ7] // put ATE into a scalar
. di ate_no_neigh
-.45811935
```

Step 6. SEE THE MAGNITUDE OF THE NEIGHBORHOOD-INTERACTIONS BIAS

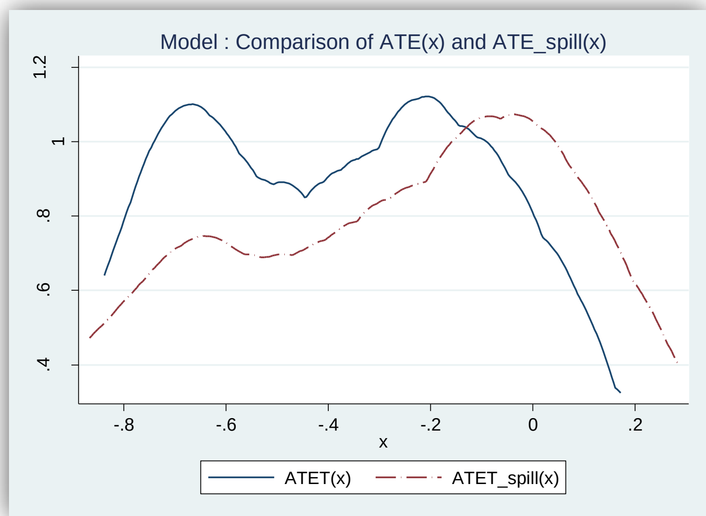
```
. scalar bias= ate_no_neigh - ate_neigh  // in level
. di bias
-.07112545
. scalar bias_perc=(bias/ate_no_neigh)*100  // in percentage
. di bias_perc
15.525528   // there is a 15% of bias due to social interaction
```

Step 7. COMPARE GRAPHICALLY THE DISTRIBUTION OF ATE(**x**), ATET(**x**) and ATENT(**x**) WITH AND WITHOUT NEIGHBOURHOOD-INTERACTION

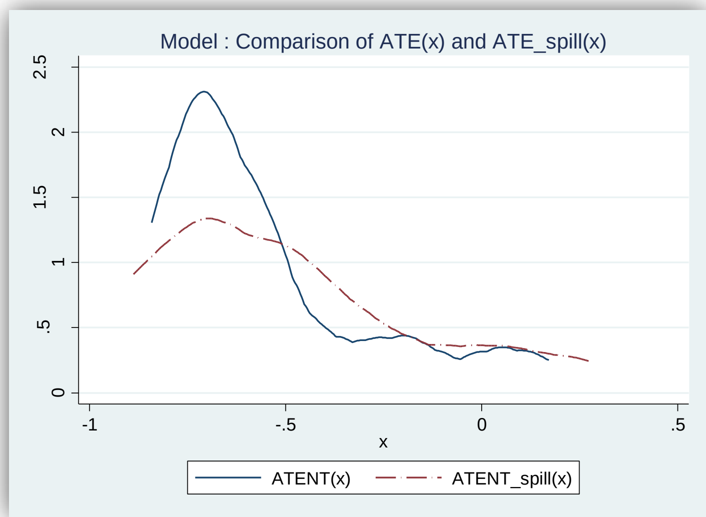
```
* ATE
twoway kdensity ATE_x , ///
|| ///
kdensity _ATE_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATE(x)" 2 "ATE_spill(x)")) ///
title("Model `model': Comparison of ATE(x) and ATE_spill(x)",
size(medlarge))
```




```
* ATET
twoway kdensity ATET_x , ///
|| ///
kdensity _ATET_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATET(x)" 2 "ATET_spill(x)")) ///
title("Model `model': Comparison of ATE(x) and ATE_spill(x)", size(medlarge))
```



```
* ATENT
twoway kdensity ATENT_x , ///
|| ///
kdensity _ATENT_x_spill ,lpattern(longdash_dot) xtitle() ///
ytitle(Kernel density) legend(order(1 "ATENT(x)" 2 "ATENT_spill(x)")) ///
title("Model `model': Comparison of ATE(x) and ATE_spill(x)", size(medlarge))
```



As a conclusion, we can state that if the analyst does not consider neighbourhood effects, she will "over-estimate" the actual effect of education on fertility of around a 15%. Furthermore, the test shows that the neighbourhood effect is relevant, as the coefficients of the neighbourhood component of regression (14) are jointly significant.

How can we interpret such a result? A possible argument can be that there is a peer-effect in choosing how many children to have by women. Indeed, as said before, the "desired" number of children for a woman does not depend only on her individual determinants ("age", for instance), but also on "what my friends do". In our sample, the existence of such a "social interaction" reduces the "effect of education on fertility" (from 0.45 to 0.38 on absolute values), by showing that fertility behaviour of more educated women (generally unconditionally less fertile) produce a *spillover* on less educated ones, by pushing them to reduce the number of children to have. This might be a typical "imitative behaviour" on the part of less educated women striving to be as much similar as possible to more educated ones. Therefore, "education" has an effect on "fertility", not only because schooling can delay the time to have a child, but also because education "triggers" imitative peer-effects.

6. CONCLUSION

This paper has presented a possible solution to incorporate *externality* (or *neighbourhood*) *effects* within the traditional Rubin's potential outcome model under conditional mean independence. As such, it generalizes the traditional parametric models of program evaluation when SUTVA is relaxed. As by-product, this work has also put forward `ntreatreg`, a Stata routine for estimating Average Treatments Effects (ATEs) when social interactions are present.

The two instructional applications to the causal effect of housing location on crime, and of education on fertility, seem to show that such approach can change significantly usual no-interaction results in those fields of social and economic contexts where externalities due to units' interaction may be pervasive.

Of course, this approach presents also some limitations, and in what follows we list some of its potential developments. Indeed, the model might be improved by:

- allowing also for treated units to be affected by other treated units' outcome;
- extending the model to "multiple" or "continuous" treatment, when treatment may be multi-valued or fractional for instance, by still holding CMI;
- identifying ATEs with neighbourhood interactions when *w* may be endogenous (i.e., relaxing CMI) by implementing GMM-IV estimation;
- trying to go beyond the potential outcomes' parametric form, by relying on a semi-parametric specification;
- providing Monte Carlo studies to check to which extent are model's results robust under different specification-errors in the weighting matrix **Ω**.

Finally, an interesting issue deserving further inquiry regards the assumption of exogeneity concerning the weighting matrix **Ω**. Indeed, a challenging question might be: what happens if individuals strategically modify their weighting weights to better profit of others' treatment outcome? It is clear that weights do become endogenous, thus yielding severe identification problems for previous causal effects. Future studies should tackle situations in which this possibility may occur.

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APPENDIX A

In this appendix, we show how to obtain the formulas of ATE and ATE(**x**) set out in (12) and (13). Then, we show how regression (14) can be obtained and, finally, we prove that Assumption 1 is sufficient for consistently estimating the parameters of regression (14) by OLS.

A1. *Formula of* ATE *with neighbourhood interactions.*
Given assumptions 2 and 3, and the implied equations in (7), we get that:

$$\text{ATE} = \text{E}(y_{1i} - y_{0i}) = \text{E}\left[\left(\mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i}\right) - \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\left[\mu_1 + \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\boldsymbol{\beta}_1 + \sum_{j=1}^{N_1}\omega_{ij}e_{1j}\right] + e_{0i}\right)\right] =$$

$$= \text{E}\left[\mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i} - \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\mu_1 + \gamma\left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\boldsymbol{\beta}_1 + \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{0i}\right)\right] =$$

$$= \text{E}\left[\mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i} - \mu_0 - \mathbf{x}_i\boldsymbol{\beta}_0 - \gamma\mu_1 - \gamma\left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\boldsymbol{\beta}_1 - \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} - e_{0i}\right] =$$

$$= \text{E}\left[\mu_1 - \gamma\mu_1 - \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_1 - \mathbf{x}_i\boldsymbol{\beta}_0 - \gamma\left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\boldsymbol{\beta}_1 - \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{1i} - e_{0i}\right] =$$

$$=\text{E}\left[\mu_1(1-\gamma) - \mu_0 + \mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) - \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\gamma\boldsymbol{\beta}_1 - \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{1i} - e_{0i}\right] =$$

$$= \text{E}\left[\mu_1(1-\gamma) - \mu_0 + \mathbf{x}_i\boldsymbol{\delta} - \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\gamma\boldsymbol{\beta}_1 - \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{1i} - e_{0i}\right] =$$

$$= \mu + \text{E}\left[\mathbf{x}_i\boldsymbol{\delta} - \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\gamma\boldsymbol{\beta}_1 - e_i\right] = \mu + \bar{\mathbf{x}}_i\boldsymbol{\delta} - \gamma\left(\sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}_j\right)\boldsymbol{\beta}_1$$

where $\mu = \mu_1(1-\gamma) - \mu_0$, $\bar{\mathbf{x}}_j = \text{E}(\mathbf{x}_j)$ and $\boldsymbol{\delta} = \boldsymbol{\beta}_1 - \boldsymbol{\beta}_0$. ■

A2. *Formula of* ATE($\mathbf{x}_i$) *with neighbourhood interactions.*
Given assumptions 2 and 3, and the result in A1, we get:

$$\text{ATE}(\mathbf{x}_i) = \text{E}(y_{1i} - y_{0i} \mid \mathbf{x}_i) = \mu + \text{E}\left[\mathbf{x}_i\boldsymbol{\delta} - \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\gamma\boldsymbol{\beta}_1 - e_i \mid \mathbf{x}_i\right] = \mu + \mathbf{x}_i\boldsymbol{\delta} - \left(\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\right)\gamma\boldsymbol{\beta}_1 +$$

$$+[\bar{\mathbf{x}}\boldsymbol{\delta} - \bar{\mathbf{x}}\boldsymbol{\delta}] + \left[\sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1 - \sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1\right] = \left(\mu + \bar{\mathbf{x}}\boldsymbol{\delta} - \sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1\right) + (\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + \sum_{j=1}^{N_1}\omega_{ij}(\bar{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1 =$$

$$= \text{ATE} + (\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + \sum_{j=1}^{N_1}\omega_{ij}(\bar{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1$$ ■

A3. *Obtaining regression (14).*

By substitution of the potential outcome as in (7) into the potential outcome model, we get that:

$$y_i = \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\sum_{j=1}^{N_1}\omega_{ij}y_{1j} + e_{0i} \right) + w\left[(\mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + e_{1i}) - \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\sum_{j=1}^{N_1}\omega_{ij}y_{1j} + e_{0i} \right) \right] =$$

$$= \left(\mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\sum_{j=1}^{N_1}\omega_{ij}y_{1j} + e_{0i} \right) + w_i(\mu_1 - \mu_0) + w_i\mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) + w_i(e_{1i} - e_{0i}) - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}y_{1j} =$$

$$= \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\mu_1 + \left(\gamma\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j \right)\boldsymbol{\beta}_1 + \gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{0i} + w_i(\mu_1 - \mu_0) + w_i\mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) + w_i(e_{1i} - e_{0i}) -$$

$$- w_i\gamma\mu_1 - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\boldsymbol{\beta}_1 - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} =$$

$$= \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\mu_1 + \underbrace{[\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{0i} + w_i(e_{1i} - e_{0i}) - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j}]}_{e_i} + w_i(\mu_1 - \mu_0) + w_i\mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) -$$

$$- w_i\gamma\mu_1 - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\boldsymbol{\beta}_1 =$$

$$= \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\mu_1 + w_i(\mu_1 - \mu_0) + w_i\mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) - w_i\gamma\mu_1 - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\boldsymbol{\beta}_1 + e_i =$$

$$= (\mu_0 + \gamma\mu_1) + w_i(\mu_1 - \mu_0 - \gamma\mu_1) + \mathbf{x}_i\boldsymbol{\beta}_0 + w_i\mathbf{x}_i(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_0) - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\boldsymbol{\beta}_1 + e_i =$$

$$= (\mu_0 + \gamma\mu_1) + w_i(\mu_1 - \mu_0 - \gamma\mu_1) + \mathbf{x}_i\boldsymbol{\beta}_0 + w_i\mathbf{x}_i\boldsymbol{\delta} - w_i\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\gamma\boldsymbol{\beta}_1 + e_i =$$

$$= (\mu_0 + \gamma\mu_1) + w_i(\mu_1 - \mu_0 - \gamma\mu_1) + \mathbf{x}_i\boldsymbol{\beta}_0 + w_i\mathbf{x}_i\boldsymbol{\delta} - w_i\sum_{j=1}^{N_1}\omega_{ij}\mathbf{x}_j\gamma\boldsymbol{\beta}_1 + e_i +$$

$$+ \left[w_i\bar{\mathbf{x}}_i\boldsymbol{\delta} - w_i\bar{\mathbf{x}}_i\boldsymbol{\delta} \right] + \left[w_i\sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1 - w_i\sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1 \right] =$$

$$= (\mu_0 + \gamma\mu_1) + w_i(\mu + \bar{\mathbf{x}}_i\boldsymbol{\delta} - \sum_{j=1}^{N_1}\omega_{ij}\bar{\mathbf{x}}\gamma\boldsymbol{\beta}_1) + \mathbf{x}_i\boldsymbol{\beta}_0 + w_i(\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + w_i\sum_{j=1}^{N_1}\omega_{ij}(\bar{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1 + e_i$$

$$= \eta + w\cdot \mathrm{ATE} + \mathbf{x}_i\boldsymbol{\beta}_0 + w_i(\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + w_i\sum_{j=1}^{N_1}\omega_{ij}(\bar{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1 + e_i$$

Therefore, we can conclude that:

$$y_i = \eta + w_i \cdot \text{ATE} + \mathbf{x}_i \boldsymbol{\beta}_0 + w_i(\mathbf{x}_i - \overline{\mathbf{x}})\boldsymbol{\delta} + w_i \sum_{j=1}^{N_1} \omega_{ij}(\overline{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1 + e_i$$

or equivalently:

$$y_i = \eta + w \cdot \text{ATE} + \mathbf{x}_i \boldsymbol{\beta}_0 + w_i(\mathbf{x}_i - \overline{\mathbf{x}})\boldsymbol{\delta} + w_i \sum_{j=1}^{N} \omega_{ij} w_j(\overline{\mathbf{x}} - \mathbf{x}_j)\gamma\boldsymbol{\beta}_1 + e_i$$

where $\eta = \mu_0 + \gamma\mu_1$, $\boldsymbol{\delta} = \boldsymbol{\beta}_1 - \boldsymbol{\beta}_0$. ■

A4. *Ordinary Least Squares (OLS) consistency*.

Under Assumption 1 (CMI), the parameters of regression (14) can be consistently estimated by OLS. Indeed, it is immediate to see that the mean of e_i conditional on (w_i; $\mathbf{x}_i$) is equal to zero:

$$\text{E}\left[\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} + e_{0i} + w_i(e_{1i} - e_{0i}) - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} \,|w_i, \mathbf{x}_i\right] =$$

$$\text{E}\left[\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} \,|w_i, \mathbf{x}_i\right] + \text{E}\left[e_{0i} \,|w_i, \mathbf{x}_i\right] + \text{E}\left[w_i(e_{1i} - e_{0i})|w_i, \mathbf{x}_i\right] - \text{E}\left[w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}e_{1j} \,|w_i, \mathbf{x}_i\right] =$$

$$\gamma\sum_{j=1}^{N_1}\omega_{ij}\text{E}\left[e_{1j} \,|\mathbf{x}_i\right] + \text{E}\left[e_{0i} \,|\mathbf{x}_i\right] + w_i\text{E}\left[(e_{1i} - e_{0i})|\mathbf{x}_i\right] - w_i\gamma\sum_{j=1}^{N_1}\omega_{ij}\text{E}\left[e_{1j} \,|\mathbf{x}_i\right] = 0$$

where $\eta = \mu_0 + \gamma\mu_1$. ■

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