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MOMENTO ANALITICO E MOMENTO CIVILE NELL'ESPERIENZA DELL'ECONOMISTA. SAGGI DI TULLIO BAGIOTTI

di
ACHILLE AGNATI *, ALDO MONTESANO ** e PIER LUIGI PORTA **

Si pubblica in questi giorni dall'Editore CEDAM un volume di saggi di Tullio Bagiotti, che intende raccogliere momenti salienti del suo lavoro di economista.

Riproducendo qui l'introduzione al volume, questa rivista vuole ricordare ai lettori la personalità scientifica del suo co-fondatore e per trent'anni direttore.

Allievi di Tullio Bagiotti, pur in stagioni e sedi diverse e con diverso rapporto di collaborazione, proponiamo con questa antologia una immagine della sua opera scientifica. Con affetto e stima per l'uomo e per lo studioso premettiamo alcune riflessioni introduttive.

Tra le numerose testimonianze seguite alla improvvisa scomparsa di Tullio Bagiotti il 28 marzo 1983, un quotidiano milanese, « l'Unità », ricordava come alla scuola di Tullio Bagiotti fossero « cresciute numerose leve di bocconiani che lo ebbero ineguagliabile e appassionato maestro di dottrine economiche ». Tullio Bagiotti infatti – proseguiva il cronista – aveva rappresentato « un'epoca feconda e lo stile di una cultura raffinata e lucidissima negli studi di economia e delle dottrine economiche ». Vogliamo qui ripercorrere brevemente il cammino dello studioso e la sua altissima lezione di rigore e di stile.

Tullio Bagiotti nasce il 16 agosto 1921 in un piccolo centro della Valtellina, Castione Andevenno. Al viaggiatore, che discende la vallata,

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l'abitato di Castione Andevenno appare poco dopo aver superato il capoluogo di Sondrio, sulla sponda destra dell'Adda, ricca, per la felice esposizione, di colture e di vigneti. Tullio è il minore dei quattro figli di una famiglia di agricoltori, coltivatori diretti, del luogo; una famiglia castionese da diverse generazioni, semplice e dedita al lavoro dei campi. Il padre, Giovanni, è ricordato come uomo probo e austero; la sua rara compostezza gli aveva guadagnato universale rispetto fra gli abitanti della comunità locale nella quale aveva assunto anche responsabilità amministrative. Una precoce intelligenza e curiosità intellettuale assai viva inducono la famiglia a decidere di avviare Tullio a studi che egli compie in buona misura privatamente.

Ufficiale degli alpini durante la seconda guerra mondiale, subisce la prigionia dopo l'otto settembre 1943. La famiglia conserva un diario del periodo di prigionia: redatto sulla base di appunti, insieme con l'amico Mario Levi, il diario è uno scarno, efficacissimo racconto dell'esperienza di quegli anni dal quale traspare un forte sentimento antifascista che non si piega di fronte ad aspre sofferenze. Si tratta di un testo straordinario, rivelatore di solida maturità umana e di cultura profonda, ben superiore a quanto il titolo di ragioniere e l'iscrizione alla Università Bocconi potessero far supporre. In realtà i *Lehrjahre* del giovane Bagiotti sono soprattutto quelli di un autodidatta divorato da una inesauribile passione conoscitiva. Il forte sentimento religioso presente nella famiglia già lo aveva indotto in primo luogo ad accostarsi alla lettura della Bibbia con eccezionale libertà spirituale. Il testo biblico doveva conservare per lui profonda attrattiva durante tutto l'arco della sua vita, anche in anni nei quali la pur vivissima apertura religiosa appare velata dai segni di un certo modernistico agnosticismo che per altro verso l'aveva fatto sostanziosamente immergere nell'opera di Nietzsche.

Il rientro in famiglia dopo la prigionia avviene, secondo il diario, il 23 agosto del '45. Ripresi gli studi all'Università Bocconi, Tullio Bagiotti si laurea con il professor Demaria nell'a.a. 1948-49 discutendo una tesi su Hermann Heinrich Gossen. La tesi prelude alle sue prime pubblicazioni e soprattutto alla celebre prima edizione italiana del volume di Gossen, seguita nel 1950 nella collana dell'Istituto di Economia diretta da Giovanni Demaria. La versione di Bagiotti viene da Einaudi e Jannaccone preferita alla francese che Einaudi era stato in predicato di pubblicare negli « Scritti rari o inediti di economisti ». Di quegli anni (1952) è pure la *Storia della Università Bocconi*, la quale contribuisce all'orientamento storico-dottrinale dei suoi studi.

Nel 1954 sposa Anna Craveri che gli sarà compagna insostituibile, intelligente e attivissima, nella vita come negli studi. Dal matrimonio nasco-

no tre figli: Luisa, Giovanni, Federico. Il clima di profonda e creativa solidarietà culturale che contraddistingue casa Bagiotti è vivo nel ricordo dei numerosi frequentatori, primi fra tutti i suoi allievi. Esso non trova espressione soltanto nella attività di animazione e direzione della RISEC, la « Rivista internazionale di scienze economiche e commerciali », della quale Tullio Bagiotti è fondatore nel 1954 con Giovanni Demaria come prosecuzione della « Rivista italiana di scienze commerciali » fondata da G.C. Colli; la collaborazione di Anna e Tullio Bagiotti si estende infatti a un'ampia attività editoriale, che culmina con la cura congiunta, per i Classici Utet, della versione italiana della *Ricchezza delle nazioni* nella edizione di Cannan in occasione del bicentenario smithiano.

L'*iter* accademico di Tullio Bagiotti lo vede nel 1954 libero docente in Storia delle dottrine economiche, alla vigilia di un anno di studi e ricerche, vincitore di una Rockefeller Fellowship, presso varie sedi statunitensi. Al rientro in Italia egli assume l'incarico di insegnamento di Storia delle dottrine economiche (incarico che manterrà sino alla fine) presso l'Università Bocconi. L'*alma mater* rappresenterà per lui costantemente, con vivissima reminiscenza personale degli anni del rettorato di Giovanni Demaria, il modello di *universitas* degli studi. Tullio Bagiotti ha sempre gelosamente conservato un altissimo concetto della vita accademica; la *universitas studiorum* si giustifica unicamente per lui come l'ambito nel quale è difesa la libertà di scegliersi i propri maestri, scelta di modelli di cultura e anche di vita e momento essenziale di qualunque esperienza di autentica dedizione scientifica. A tali concetti egli attribuiva un significato duraturo, rifuggendo apertamente da ogni possibile giustificazione sociologica della *trahison des clercs* nelle sue diverse forme.

Nella seconda metà degli anni cinquanta egli insegna la medesima disciplina anche a Pavia e a Padova, insieme a Politica economica ed Economia politica. Ternato al concorso di Catania nel 1962 per Economia politica, egli è chiamato come straordinario della disciplina nella Facoltà di Giurisprudenza dell'Università di Padova dal 1 novembre 1962; a Padova, dove occupa così la cattedra che era stata di Marco Fanno, egli è altresì Direttore dell'Istituto di Scienze economiche dai primi anni sessanta sino al trasferimento. È nel 1976, infatti, che egli si trasferisce alla cattedra di Economia politica, alla quale è chiamato come primo titolare, nella Facoltà di Scienze politiche dell'Università di Milano. Socio corrispondente dell'Istituto Lombardo Accademia di Scienze e Lettere dal 1964, egli diviene più tardi, nel 1980, socio corrispondente della Accademia Nazionale dei Lincei.

I saggi raccolti nel presente volume sono divisi in tre parti che riflettono tematiche di fondo dell'opera scientifica di Tullio Bagiotti. La prima

parte è dedicata ai grandi protagonisti della teoria economica. La seconda parte riproduce alcune significative espressioni del suo pensiero economico. La terza parte, infine, raccoglie qualcuno dei numerosissimi interventi di Tullio Bagiotti su questioni economiche di attualità, interventi che hanno segnato, nei contenuti e nello stile, il 'momento civile' della sua attività.

L'apertura del volume è naturalmente riservata a Gossen, con il saggio commemorativo – replicato in versione tedesca sulla « Zeitschrift fuer Nationaloekonomie » nel 1957 – pubblicato in origine sul « Giornale degli economisti » nel 1955. Il saggio riflette l'esperienza di traduttore e curatore della *Entwickelung*. Nicholas Georgescu-Roegen, nel saggio introduttivo alla prima edizione inglese della stessa opera – pubblicata assai più recentemente, nel 1983, dalla MIT Press – tributa giusto riconoscimento, nella letteratura gosseniana, al lavoro originario di Bagiotti per l'edizione italiana e alla sua « introduzione molto ben riflettuta e assai informativa ». Anche rispetto agli altri protagonisti qui ricordati, così come per il caso di Gossen, l'opera di rivisitazione nasce per Bagiotti in simbiosi con l'attività editoriale di cura e traduzione. Bagiotti ha infatti dedicato buona parte della sua attività scientifica ai maggiori momenti dell'analisi economica [1]. In questa attività ha avuto indubbiamente parte preponderante, seppure non esclusiva, la letteratura tedesca: oltre a Gossen, Auspitz & Lieben, Launhardt, Mises, Boehm-Bawerk, Wieser. A questi autori si accompagna una diversificazione di interessi che lo conduce ad esempio alla edizione del 'Giornale' paretiano (1958), alla analisi di ricostruzione tematica della esperienza dello « Schmollers Jahrbuch » (1954), a Henry George, a Frank Knight, a Joseph Schumpeter. Un filone di studi di particolare significato collega Bagiotti con il pensiero economico italiano: esso culmina probabilmente con il notevole saggio su *L'economia di Alessandro Manzoni* (1975), ma tocca in tempi diversi molti altri autori tra i quali Romagnosi, Cattaneo, Fanno, Del Vecchio, Demaria.

Più recente (1976) è l'interesse editoriale riservato a Smith con l'edizione Utet della *Ricchezza*. Ma Smith occupa un posto di rilievo in tutta la sua produzione, com'è palese nella monografia *Il profitto* (1965), pure largamente assorbita da altri interessi. La sintesi smithiana ha speciale valore paradigmatico nella concezione sviluppata da Bagiotti (irrobustita dal 'modo' di J.B. Clark), concezione che anticipa la rivisitazione storiografica di Smith, nella critica più recente, intesa a liberarne l'immagine dalle strettoie di opposti stereotipi di taglio liberalcapitalistico, da un lato, o mutuati dal

[1] È il motivo conduttore dell'*in memoriam* alla Società Italiana degli Economisti di A. AGNATI, *Tullio Bagiotti: una commemorazione congiuntiva*, in « Atti » della XXIV Riunione Scientifica tenutasi a Roma nei giorni 10-11 novembre 1983: AA.VV., *I limiti della politica economica*. Milano: Giuffrè, 1985, pp. 271-87.

classicismo economico di ispirazione marxista, dall'altro. Il modo tenuto da Smith – si legge nel saggio di Bagiotti (1977) sulla *Perdurante validità* dell'opera di questi – è quello della letteratura eziologica settecentesca. « Adesso l'eziologia è un ramo della medicina interessato alle cause delle malattie. Nel mondo greco e latino e ancora, con forte reviviscenza, nella letteratura del Settecento col suo interesse alla storia ragionata e alla storia naturale, la letteratura era eziologica, cioè sensibilizzata sulla ricerca delle origini o cause delle città, degli ordinamenti, dei miti, dei valori e delle esperienze delle comunità ». Ma – egli prosegue – non solo « la specializzazione ha dannato questo modo eziologico », ma la stessa nozione corrente di 'scuola classica' in economia ne eclissa il significato: ad esempio « Ricardo, nei cui *Principi* lo spirito di geometria si è tanto riconosciuto e compiaciuto, deve essere considerato come una vera disgrazia eziologica ».

L'interpretazione e lo studio di Smith si saldano fortemente con il continuo interesse di Tullio Bagiotti per il tema del determinismo e indeterminazione in Economia politica, avendo il Maestro Demaria impostato nel 1932 la sua dinamica sul principio di indeterminazione mutuato dalla fisica di Heisenberg (1927). Qui ritroviamo un momento unificante della sua produzione: vorremmo in particolare ricordare al riguardo l'esame critico e la sintesi che egli propone di una « discussa categoria economica » ne *Il profitto*, monografia che occupa un posto centrale tra i suoi studi. Più in generale, egli amava ripetere che l'economia è una disciplina dell'anima ed è un errore legarne le origini, in verità remotissime, alle forme dello scientismo e della modernità. Forse l'espressione più caratteristica del tema del determinismo e indeterminazione nella versione cara a Tullio Bagiotti, può essere ritrovata nel dilemma boembawerkiano di 'legge' o 'forza economica', dilemma che è in realtà l'espressione di una simbiosi fondamentale in Economia politica, come ricorda assai opportunamente un significativo recente volume di Anna Pellanda [2]. Così, nella sua celebre introduzione a Boehm-Bawerk (1956), Tullio Bagiotti scrive del « lascito spirituale del loico che rivela finalmente non essere stato insensibile alle gravi riflessioni sollevate contro la certezza della legge economica » e conclude la sua parabola con una argomentazione – *Macht oder oekonomisches Gesetz* – « forse non a torto ritenuta un modello del genere ».

Nella produzione di Tullio Bagiotti il tema trova numerosi altri riscontri, che vanno dalla censura alle versioni meccanicistiche o 'idrauliche' del

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[2] PELLANDA A., *Sviluppo economico: 'legge' di produzione industriale o 'forza' di terziario?. Il principio di complementarità nella metodologia, storia e analisi economica*. Padova: Cedam, 1989.

pensiero keynesiano alla « indebita saccenteria » – come si legge nella introduzione alla *Ricchezza* – di un marxismo di maniera che pretende di servirsi di Smith per poi liquidarlo come portavoce della borghesia. Tullio Bagiotti – al pari di un lignaggio di profondi conoscitori di Marx alla maniera di Schumpeter – non è mai stato marxista e questo probabilmente gli ha meglio consentito di cogliere la ricchezza e la fecondità del pensiero di Marx anch'esso irriducibile a formule 'idrauliche'.

Comprensibilmente, allorché Tullio Bagiotti fa la sua prima comparsa nelle aule della Facoltà di Scienze politiche di Milano nel 1976, le contestazioni degli studenti, notoriamente preda di un preteso progressismo marxista costruito su facili slogan, sono assai vivaci. Il linguaggio e lo stile di un accademico appaiono inevitabilmente curiosi e irreali alla folla di giovani e meno giovani intenti a consumare il rito collettivo della corsa ai beni posizionali. Il rito ha, proprio in quegli anni, raggiunto il parossismo: abbondano verbosità minacciose e truculente cui non di rado fanno seguito i fatti in un clima di intimidazione e di violenza. Le aule universitarie della giovane Facoltà non hanno nulla di accademico; allo sguardo possono, piuttosto, richiamare il moto variopinto e chiassoso di una stazione ferroviaria con relativi assalti alle carrozze. Alcuni tra i maestri sono contagiati dalla febbre del democratismo giacobino; non pochi – sarebbe ipocrita negarlo – pensano più pragmaticamente a salvare la pelle e la carriera. Un episodio diventa il segno di uno stile: tra il clamore delle masse, Tullio Bagiotti – cui è tolta la parola – con calma scrive sulla lavagna una frase: « Segui il tuo corso, e lascia dir le genti! » e, alla riga successiva, tra lo stupore della massa e dei suoi leader, « Marx ». Nessuno di coloro naturalmente teneva in qualche considerazione una massima tanto individualistica; ma nessuno aveva neppure la minima idea che Marx stesso – indubbiamente buon profeta nell'anticipare la distanza tra sé stesso e la schiera dei futuri marxisti – avesse scelto – il 25 luglio 1867 – proprio le parole del grande fiorentino a concludere l'introduzione della prima edizione del *Capitale*.

Bagiotti ha posseduto una conoscenza impareggiabile e straordinaria della maggiore letteratura economica e di essa era intessuto tutto il suo discorso, assai fertile soprattutto nella conversazione accademica pur spesso limitata a quel ristretto gruppo di allievi e studenti al quale abbiamo avuto il privilegio di appartenere anche come suoi assistenti. A queste qualità si accompagnava in lui quell'evidente impaccio nel ragionamento formalizzato che ha contraddistinto non pochi economisti del suo calibro. Il punto è delicato anche perché coinvolge subito la forte tentazione di radicalizzare il discorso sul piano metodologico. Su questo aspetto sembra opportuno rilevare come Tullio Bagiotti abbia certamente creduto che il clima intellettuale

del dibattito scientifico contemporaneo in Economia politica, pur risultando dall'interagire di una molteplicità di componenti, sia stato eccessivamente ed indebitamente influenzato dalla concezione epistemologica neopositiva. Sarebbe tuttavia profondamente fuorviante credere che egli prefigurasse una sorta di 'rivincita' in astratto del metodo storico o una messa al bando della formalizzazione. Egli non è stato critico nei confronti degli *strumenti* concettuali inerenti per sé presi, bensì della pretesa, non di rado avanzata, di completa sovrapposizione tra procedura formalizzante e processo cognitivo. Il rifiuto di una tal sovrapposizione gli è derivato, crediamo, soprattutto dal percepire in esso una troppo facile scorciatoia alla *pretesa* del sapere; quella sovrapposizione, una volta introdotta attraverso i canoni neopositivi, si trasforma inarrestabilmente in una insidiosa barriera rispetto a quello che Tullio Bagiotti considerava uno dei massimi doveri dello studioso, ossia l'esercizio e la crescita costante nella autocoscienza socratica rispetto alla propria condizione naturale di ignoranza. Qui gli aspetti, ad esempio, di profonda compenetrazione tra teoria e storia, così come essa traspare dai suoi scritti, diventano straordinariamente attuali in un clima intellettuale qual è l'attuale, nel quale la riflessione metodologica non può non percepire la crisi inarrestabile – testimoniata ormai da un'amplissima letteratura – dell'orizzonte neopositivo.

Potrà oggi sembrare a taluno ingenuità la costante vigile attenzione che Tullio Bagiotti dedica alla economia corrente attraverso gli editoriali della RISEC [3]. Si tratta, in realtà, a chi voglia positivamente considerare questo aspetto, della coerenza di un chierico. Vi è in questo quella piena realizzazione della confluenza tra momento analitico e momento civile dell'economista, che abbiamo voluto qui riprendere da un suo titolo e che Giampiero Landenna ha efficacemente messo in luce a suo tempo nella commemorazione tenuta all'Istituto Lombardo [4]. La viva attualità delle tematiche toccate dal Maestro è così evidente che sembra opportuno non aggiungere altre parole alle sue per chi vorrà leggerle. Naturalmente, ad esempio, la lamentela contro gli alti tassi di interesse come la preoccupazione per le dinamiche del

[3] Se ne può trarre – sinteticamente – sostanza, modo, giudizio da A. AGNATI, *Gli editoriali di Tullio Bagiotti*, in AA.VV., *Studi in memoria di Tullio Bagiotti*, a cura di A. Agnati, D. Cantarelli, A. Montesano. Padova: Cedam, 1988, pp. 1-32.

[4] Commemorazione tenuta dal m.e. Giampiero Landenna, in Rendiconti dell'Istituto Lombardo, Accademia di Scienze e Lettere, vol. CXIX (1985), Milano, 1988, pp. 147-52. Anche in *Studi in memoria di Tullio Bagiotti*, pp. lv-lix, cit., Tullio Bagiotti prediligeva in particolare il termine 'civile' tratto dall'insegnamento di Genovesi e successivamente ripreso, con diversa accentuazione, da Romagnosi.

debito pubblico sono oggi, a distanza di quasi vent'anni da taluni di quei commenti, diventate moneta corrente. Il consenso sulla 'rivincita del mercato' sembra oggi aver conquistato anche i più ostinati artefici dell'economia regolata, molti dei quali paiono avere frettolosamente riscoperto l'efficienza produttiva e la sovranità del consumatore. La coerenza temporale non è certo sempre considerata una virtù fra i commentatori economici, anche quando essi vestono panni accademici, giacché il loro commento non di rado si prefigge come scopo prevalente quello di conquistare l'opinione e di influire sugli operatori. Si tratta, pur con rilevanti eccezioni, di commento che è esclusiva opera di *politici*, ossia di quegl'insidiosi e astuti animali che, appunto, sanno – com'è nella loro natura – parlare e agire esclusivamente sulla base delle circostanze del momento. Nelle pagine che presentiamo vi è un contributo diverso. L'economia di Tullio Bagiotti appare fondata sopra la concezione di un mercato di 'civile' concorrenza. Egli crede, alla Einaudi, nella dialettica positiva delle forze sociali come testimoniano gli interventi sugli aspetti stimolatori dell'azione organizzata del lavoro. Non crede invece nel sindacalismo al servizio di una indebita sovranità del lavoratore, foriera di corporative difese di posizioni di rendita. Così pure egli non crede nelle false sicurezze dei programmatori né nello stato quando nazionalizzatore quando imprenditore; e neppure nella falsificazione monetaria e nei vincoli valutari imposti a quei 'cittadini della gleba' che tutti siamo sempre in procinto di diventare; neppure crede nella possibile trasformazione del debito in festa collettiva. L'elenco è in realtà un coacervo di illusioni, in buona misura caratteristicamente italiane; è sin troppo facile a posteriori riconoscere in quelle illusioni, troppo a lungo alimentate, il risultato e insieme il piedestallo del predominio di una classe politica, nel senso di Gaetano Mosca, una classe politica tipicamente italiana che ha ricompreso per decenni governo e opposizione e dalla quale l'Italia d'oggi tenta a fatica di liberarsi.

Si rileggano le censure a un Ministro del tesoro divenuto monopsonista del credito e i richiami alla attualità dei canoni morali contro l'usura, ripresi anche da una pastorale dei vescovi tedeschi del 1980, usura oggi praticata prevalentemente attraverso l'indebitamento pubblico; o, ancora, le critiche – precise e severe – alla disinvoltura corrente in materia monetaria, prive di indulgenza verso il nostro stesso Istituto di emissione. Vi è una efficace espressione, coniata da Tullio Bagiotti, che esprime forse al meglio la critica complessiva ch'egli rivolge a quella 'cultura dell'illusione' che ha dominato l'opinione pubblica, specie italiana, e che da noi è stata spregiudicatamente utilizzata dalle sinistre come dai governi. Egli parla – e introduce il termine, significativamente, anche nel *Corso di Economia* (1977) a beneficio dei suoi

studenti coi quali egli ha sempre avuto un intenso rapporto di docente ed educatore – di « prepotere del commensalismo », secondo il titolo di una memorabile comunicazione all'Istituto Lombardo dell'inizio degli anni ottanta. È il prepotere del commensalismo che contrassegna il rapporto oppressivo tra governanti e governati sino a inaridire le sorgenti del progresso; snatura le funzioni dei parlamenti dimentichi dell'unica funzione che può giustificarli, ossia quella di farsi garanti di un principio di 'no taxation without representation'; deforma le ragioni economiche dell'interesse svincolandolo dal necessario legame con il profitto; forza ogni aspetto della socialità oltre qualunque ragionevole vincolo di produzione e di reddito. Questi – in Bagiotti – i dati essenziali di una concezione dell'impegno civile dell'economista. L'efficacia della parola dell'economista, in questo senso del termine, non può *mai* essere esaurita. Tanto meno sembra poterlo essere, se guardiamo dentro casa nostra, in questo caso e nella presente situazione, poiché la parola dell'economista sembra qui e oggi suggerire con speciale evidenza criteri di valutazione sulle credenziali di coloro che si dichiarano a un tempo vergini e innovatori, aspirando a reggere le sorti della repubblica. A costoro l'economista impartisce una lezione di umiltà sino a renderne scoperte le false pretese, con una iniziativa che solo superficialmente potrebbe essere considerata sterile o addirittura estranea ai suoi compiti.

ANALYTIC AND CIVIC ASPECTS IN THE ECONOMIST'S EXPERIENCE. ESSAYS IN POLITICAL ECONOMY BY TULLIO BAGIOTTI

Just in these days is being published by CEDAM a volume of essays by Tullio Bagiotti which intends to gather the most important moments of his work as an economist.

By reproducing the introduction to the volume, this review wants to remind its readers the scientific personality of its co-founder and editor for thirty years.

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A NEW MODEL OF AN INHOMOGENEOUS, MULTIFACTOR PRODUCTION FUNCTION

by
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and BRUNO GIACOMELLO ***

1. *Introduction*

The production function whose prominent properties are set out in the present paper, is the result of the extension of a two-factor function, called (perhaps with a shade of immodesty) CMC after its authors' names, to a three-factor one [1]. In our opinion such an extension is significant on two grounds at least. Firstly, as far as the analytical treatment and the economic implications of the model are concerned, it introduces a qualitative change and not only a quantitative one. The most important example of such a change is given by the different version of the (now) partial elasticities of substitution. Secondly the model becomes undeniably more realistic. Let's think not only of two kinds of labour or fixed capital, but also of natural resources, a factor that is becoming more and more important, to which economists are rightly going to pay growing attention.

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Sections 3 and 4 are to be attributed to B. Giacomello, Section 5 to T. Di Fonzo, and the remaining ones to D. Cantarelli.

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[1] See CANTARELLI et al. (1990).

It could be objected, however, that after the success of the translog production and cost functions, the introduction of a new function might be regarded as strange, superfluous or, at the very least, open to debate. It is a well-known fact that translog functions have made it possible to carry out an econometric estimate of the interrelated demands for (more than two) factors of production and their elasticities of substitution [2].

But every success has its costs, and this was no exception. In our opinion, the most important ones are: a) the possible existence of "non-economic regions" in the production hyperspace; b) the lack of "duality" between cost and production functions; c) the need to do away with the simpler (and convenient) assumptions of homogeneity and homotheticity of the production function [3]. We firmly believe that from the "history" of the production function it can be concluded that goals of generality, flexibility and realism can *never* be wholly and jointly accomplished. For this reason we feel sure that there is still room, both now and in the foreseeable future, for new productions which show their ability reasonably to pursue – and at least partly achieve – the aforesaid objectives.

The paper is divided into six Sections. After Section 1 (Introduction), Section 2 illustrates the chief analytic properties of the function and their economic implications. The simpler two-factor version is used only when it is deemed conducive to greater clarity. In Section 3 we discuss aspects relating to the different versions of the partial elasticities of substitution, their properties and meanings. Section 4 presents the cost function implicit in the CMC and its variants. In Section 5 we present a few results from econometric tests worked out on data from various sources (and which, due to their simplicity, we have called exercises). Section 6 contains the conclusions we may draw at this stage of our research and suggests several paths along which our work may proceed.

2. *Properties of the* CMC *Model (Complete Function and Variants)*

The CMC production function is "generated" through the generalization of the expansion path equation of a relatively simple function such as the Cobb-Douglas (or CES), by giving up the principle-hypothesis that it can be exclusively rectilinear [4]. Given the three factor (but this also applies

[2] The chief applications of translog functions were made at macroeconomic level (BERNDT and WOOD, 1975). For applications at an intersectorial level in the context of general equilibrium, see JORGENSON (1984) and BERNDT (1991).

[3] See CHUNG (1977), BERGSTRÖM and PANAS (1992) and KIM YOUNG (1992).

[4] See CANTARELLI et al. (1990).

to four or more) Cobb-Douglas function $y = \gamma x_1^{\alpha} x_2^{\beta} x_3^{\delta}$, by transforming the relationships

$$\frac{\partial y}{\partial x_1} \Big/ \frac{\partial y}{\partial x_2} = \frac{\alpha}{\beta} \frac{x_2}{x_1}, \qquad \frac{\partial y}{\partial x_1} \Big/ \frac{\partial y}{\partial x_3} = \frac{\alpha}{\delta} \frac{x_3}{x_1}, \qquad \frac{\partial y}{\partial x_2} \Big/ \frac{\partial y}{\partial x_3} = \frac{\beta}{\delta} \frac{x_3}{x_2}$$

into

$$\frac{\partial y}{\partial x_1} \Big/ \frac{\partial y}{\partial x_2} = \frac{\alpha}{\beta} \frac{x_2^m}{x_1^n}, \qquad \frac{\partial y}{\partial x_1} \Big/ \frac{\partial y}{\partial x_3} = \frac{\alpha}{\delta} \frac{x_3^q}{x_1^n}, \qquad \frac{\partial y}{\partial x_2} \Big/ \frac{\partial y}{\partial x_3} = \frac{\beta}{\delta} \frac{x_3^q}{x_2^m}$$

the production function, with three factors, say labour (L), capital (K) and energy (E)

$$Y = \gamma \exp\left(\frac{\alpha}{1-n} L^{1-n} + \frac{\beta}{1-m} K^{1-m} + \frac{\delta}{1-q} E^{1-q} \right) \qquad (1)$$

$$\gamma > 0$$
$$n, m, q > 1$$
$$\alpha, \beta, \delta > 0$$

can be obtained. We call expression (1) *complete* version of the CMC production function. "Variants" of (1) can be obtained by assuming that one or a pair of factors appear in a "Cobb-Douglas" form [5]. Moreover, all the exponents may in turn be equal or equal in pairs (but always > 1) and no possibility may be rejected *a priori*. Taken as a whole, the complete CMC function and its variants are called "CMC model".

From an analytical point of view the complete function (1) is of course the most complex case. Consequently all its general and specific properties must be made explicit. The general properties of (1) are as follows: for each of its arguments, it is a transcendent, continuous, monotonic, strictly increasing and almost concave function, with first and subsequent partial derivatives that may always be calculated; it is (and remains) inhomogeneous, although it can be homothetic (i.e., when $n = m = q$) and separable [6]. Lastly, it may be inverted, and for this reason it is possible to determine – implicitly, though sometimes explicitly – its dual cost function. Since its exponents are negative, it tends to asymptotes. Parameter γ, which has the same logical dimension as Y, is the limit to which the latter

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[5] That is when a factor, say L, appears in the form $\alpha \ln L$ instead of $\frac{\alpha}{1-n} L^{1-n}$.

[6] Separability may be checked for all variants of the CMC model. As it is well known, this is a very useful property. In a three factor function, weak and strong separability actually coincide.

tends for L, K, $E \to +\infty$. Parameters n, m and q are dimensionless; α, β and δ have L, K, and E dimension respectively. Their dimensionality does not create any limits or disadvantages.

For fixed K and E,

$$\lim_{L \to +\infty} Y = \gamma \exp\left(\frac{\beta}{1-m} K^{1-m} + \frac{\delta}{1-q} E^{1-q}\right) = \hat{Y}_L$$

and similar expressions hold for K and E.

To illustrate the chief specific properties of the function, wherever possible we shall refer only to factor L, since these properties are also applicable, *mutatis mutandis*, to K and E. As far as marginal product (MP) and average product (AP) are concerned, we have

$$\frac{\partial Y}{\partial L} = \alpha \frac{Y}{L^n} > 0; \quad \frac{\partial^2 Y}{\partial L^2} = \alpha Y L^{-(n+1)} [\alpha L^{1-n} - n] \gtrless 0$$

$$\text{with } L \lessgtr (\alpha/n)^{1/(n-1)} = A \qquad (2)$$

$$\frac{\partial (Y/L)}{\partial L} = Y(\alpha L^{1-n} - 1)/L^2 \gtrless 0 \qquad \text{with } L \lessgtr \alpha^{1/(n-1)} = B \qquad (3)$$

Furthermore, $\frac{\partial Y}{\partial L} = \frac{Y}{L}$ when $L = B$, and B is strictly greater than A for $n > 1$. Both A and B, quantities of L for which, respectively, MP reaches its maximum value and Ap equals MP, are determined only by parameters α and n. Formulae (2) and (3) show that levels A and B rise, α being fixed, when n increases, and, n being fixed, when α increases. The values and patterns of MP and AP are also determined, *coeteris paribus*, by the levels of α and n. With α costant and $n_1 < n_2$, functions MP_2 and AP_2 have higher maximum values than MP_1 and AP_1, above which they will certainly increase for a certain interval of L values. However, it may be that, from a certain value of L onwards, MP_2 and AP_2 pass *beneath* MP_1 and AP_1 respectively [7]. With n constant and $\alpha_1 < \alpha_2$, functions MP_2 and AP_2 have *lower* values than MP_1 and AP_1. Production functions with high values of n (m and q) at the parity of α, β and δ, attain values of efficiency as high as those with lower n (m and q) which, however, can also decrease very swiftly. Parameters α, β and δ express instead the intensity of technolog-

[7] By resolving the system between two functions like MP_1 and MP_2, it is easy to check that the value of L (if it exists) for which $MP_1 = MP_2$, is determined only by n_1 and n_2.

ical or institutional obstacles or constraints to production processes and it is therefore to be hoped that these have moderate values.

It should be pointed out that

$$\frac{\partial^2 Y}{\partial L \partial K} = \alpha\beta \frac{Y}{L^n K^m} > 0; \quad \frac{\partial^2 Y}{\partial L \partial E} = \alpha\delta \frac{Y}{L^n E^q} > 0;$$
$$\frac{\partial^2 Y}{\partial K \partial E} = \beta\delta \frac{Y}{K^m E^q} > 0$$

The partial elasticity of production or *passus* coefficient [8] is given by

$$\varepsilon_{Y/L} = \frac{\partial Y}{\partial L} \frac{L}{Y} = \alpha L^{1-n} > 0; \lim_{L \to 0} \varepsilon_{Y/L} = +\infty; \lim_{L \to +\infty} \varepsilon_{Y/L} = 0$$

$\varepsilon_{Y/L} = n$ for $L = A$; $\varepsilon_{Y/L} = 1$ for $L = C$ and similarly for K and E. The total *passus* coefficient

$$\varepsilon_Y = \alpha L^{1-n} + \beta K^{1-m} + \delta E^{1-q} \tag{4}$$

may assume value 1 only for values of $L > C$ and for values like C referring to K and E.

With two factors of production, and putting $K = f(L)$,

$$\frac{dK}{dL} = -\frac{\alpha}{\beta} \frac{K^m}{L^n} < 0; \quad \frac{d^2K}{dL^2} = -\frac{\alpha}{\beta} \frac{mK^{m-1} \cdot \frac{dK}{dL} L^n - K^m n L^{n-1}}{L^{2n}} > 0 \tag{5}$$

and for this reason the isoquants are always diminishing and convex [9]. The equation of isoquants, with a given $\bar{Y}$, that is

$$K = \left[\frac{1-m}{\beta} \ln \frac{\bar{Y}}{\gamma \exp\left(\frac{\alpha}{1-n} L^{1-n} \right)} \right]^{1/(1-m)} \tag{6}$$

enables us to check that the isoquant does not exist for each $L > 0$, but only for

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[8] The Latin term which S. Carlson proposed over fifty years ago seems more appropriate, and not only from an aesthetic point of view.

[9] If we look at (5), it is clear that the MRTS of the complete CMC is a pure, dimensionless number.

$$L > \left[\frac{\alpha}{(1-n)\ln(\bar{Y}/\gamma)}\right]^{1/(n-1)} = \check{L} \tag{7}$$

A symmetric result, i.e. a $\check{K}$ value, is obtained if we assume $L = g(K)$. This means that each isoquant is located between two asymptotes which are parallel to the L and K axes and that production of a given $\bar{Y}$ requires minimum (or "limitational") quantites $\check{L}$ and $\check{K}$. It is immediately evident that this property of the isoquants can affect the elasticity of substitution.

Continuing to focus on two factors, with K on the ordinate and L on the abscissa, the expansion path, given by

$$K = \left(\frac{\beta}{\alpha}\frac{p_L}{p_K}\right)^{1/m} L^{n/m} \tag{8}$$

is increasing and rectilinear with $n = m$, concave towards the top with $n > m$ and concave towards the bottom with $n < m$.

Extension to three factors presents no difficulties. The isoquants are surfaces in $\mathbb{R}^3$ and if, for example, they are made explicit with regard to K, with a given $\bar{Y}$, we obtain

$$K = \left[\frac{1-m}{\beta}\ln\frac{\bar{Y}}{\gamma\exp\left(\frac{\alpha}{1-n}L^{1-n} + \frac{\delta}{1-q}E^{1-q}\right)}\right]^{1/(1-m)} \tag{9}$$

and consequently the isoquant of $\bar{Y}$ exists for

$$L > \frac{1-n}{\alpha}\ln\frac{\bar{Y}}{\gamma\exp\left(\frac{\delta}{1-q}E^{1-q}\right)} \tag{10}$$

So in $\mathbb{R}^3$ the sections of isoquants with planes $L = \text{const.}$, or $K = \text{const.}$, or $E = \text{const.}$, are curves like the isoquants in the two-factor case.

Still in $\mathbb{R}^3$, total cost function C, assumed in its given value $\bar{C}$,

$$\bar{C} = p_L L + p_K K + p_E E \tag{11}$$

is the equation of a plane. The expansion path, which is the locus of points in which the isoquants have (11) as tangent plane, is given and defined by the three parametric equations

$$\alpha Y L^{-n} = p_L; \quad \beta Y K^{-m} = p_K; \quad \delta Y E^{-q} = p_E \tag{12}$$

These enable us to write the equation of the expansion path in Cartesian form, as a line of intersection between two surfaces, i.e.:

$$\begin{cases} K = \left(\dfrac{\beta p_L}{\alpha p_K}\right)^{1/m} L^{n/m} \\ E = \left(\dfrac{\delta p_L}{\alpha p_E}\right)^{1/q} L^{n/q} \end{cases} \tag{13}$$

Once again the values of n, m, q and their ratios determine concavity or convexity of the expansions path in relation to the axes.

Should one or two factors appear in a "Cobb-Douglas" form, i.e. as L^{α}, K^{β} or E^{δ} [10], variations in the function's properties are as follows:

- γ is no longer the upper limit which Y attains in approaching to $+\infty$ of the factors' quantities. An asymptote such as $\hat{Y}_L$, $\hat{Y}_K$ or $\hat{Y}_E$ exists only for factors expressed as power of e;
- each "Cobb-Douglas" factor presents *MP*, *AP* and partial *passus* coefficient belonging to the aforesaid function;
- for each "Cobb-Douglas" factor there is no "limitational" quantity like $\check{L}$, $\check{K}$ or $\check{E}$, and the isoquant has its own Cartesian axis as asymptote;
- the concavity of the expansion path always faces the Cartesian axis of each "Cobb-Douglas" factor.

3. *The Partial Elasticities of Substitution*

As experts on the subject know, 25 years ago M. Morishima, in a paper written in Japanese, supplied a new, personal definition and formula of the elasticity of substitution for a multifactor function, that was intended to offer "an economically meaningful interpretation" of σ and therefore an alternative to the well known R.G.D. Allen's formula [11]. Its novelty lies in the fact that, given an r factor production function and considering every pair of factors i and j, two elasticities $\sigma_{ij}^{M} \neq \sigma_{ji}^{M}$ have to be calculated, as

[10] In such cases, n, m or q disappear and α, β or δ become dimensionless exponents.

[11] See KUGA and MUROTA (1972, p. 285). Morishima defined σ as the "proportionate change in the ratio of the amounts of the factors divided by such a proportionate change in the marginal rate of substitution of x_j for x_i as to leave all other marginal rates of substitution and the output levels intact".

opposed to Allen's one, that is $\sigma_{ij}^{A} \equiv \sigma_{ji}^{A}$. It is to be noted that Morishima and others assumed functions with constant σ_{ij} [12]. With r productive factors, $2r$ Morishima elasticities must therefore be calculated.

Although one can well appreciate that an exhaustive comparative analysis of the σ_{ij}^{A} and σ_{ij}^{M} for the CMC model calls for a specific (and lengthy) paper, here we may provide brief yet definitive indications [13].

To this end it is more useful than ever to start out with two factors (x_1 and x_2) and to adopt the complete CMC.

If we apply Allen's complete formula, as required by an inhomogeneous function,

$$\sigma_{12}^{A} \equiv \sigma_{21}^{A} \frac{(x_1 f_1 + x_2 f_2)}{x_1 x_2} \frac{A_{12}}{\Delta} \tag{14}$$

and Morishima's formulae

$$\sigma_{12}^{M} = \frac{f_2}{x_1} \frac{A_{12}}{\Delta} - \frac{f_2}{x_2} \frac{A_{11}}{\Delta} \qquad \sigma_{21}^{M} = \frac{f_1}{x_2} \frac{A_{21}}{\Delta} - \frac{f_1}{x_1} \frac{A_{22}}{\Delta} \tag{15}$$

where f_1 and f_2 are the first partial derivatives, Δ the bordered Hessian determinant, $A_{12} \equiv A_{21}$, A_{11} and A_{22} the three co-factors, with $x_1 = L$ and $x_2 = K$, we obtain

$$\sigma_{LK}^{A} \equiv \sigma_{LK}^{M} \equiv \sigma_{KL}^{M} = \frac{\alpha K^{m-1} \beta L^{n-1}}{m\alpha K^{m-1} + n\beta L^{n-1}} \tag{16}$$

One first result is that, with two factors, *even* in an inhomogeneous function with variable σ, identity exists between Allen and Morishima versions; consequently, this identity is not restricted to linear homogeneous functions with constant σ [14]. Remembering that σ is a value, constant or variable, which is valid along any given isoquant with Y constant, $\bar{Y}$, it is easy to check that

$$\lim_{L \to \bar{L}^{+}} \sigma = 1/m; \quad \lim_{L \to +\infty} \sigma = 1/n; \quad \lim_{K \to \bar{K}^{+}} \sigma = 1/n; \quad \lim_{K \to +\infty} \sigma = 1/m \text{ [15]} \tag{17}$$

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[12] See KUGA and MUROTA (1972), MUROTA (1977) and KUGA (1979). For what we know, these authors were the first to write a paper on the subject in English.

[13] The material for this paper is nearly ready. We believed that it was essential to examine Morishima's original paper and therefore we arranged for the paper to be translated.

[14] The papers by Kuga and Murota quoted in note (10) only deal with the case of constant σ_{ij}.

[15] In calculation of limits in (17) it should be remembered that, if $L \to +\infty$, automatically $K \to \bar{K}^{+}$ and vice-versa.

Values $1/n$ and $1/m$ constitute the limits within which σ can vary and they may therefore be called *limit elasticities*. Obviously, if $m = n$, $\sigma = 1/n = = \text{const}$. From (16) it may also be seen that

$$(\partial\sigma/\partial L) \gtrless 0 \text{ with } m \gtrless n; \quad (\partial\sigma/\partial K) \gtrless 0 \text{ with } n \gtrless m$$

If one of the factors – K, for example – appears in "Cobb-Douglas" form, (16) is reduced to

$$\sigma^A_{KL} \equiv \sigma^A_{LK} \equiv \sigma^M_{KL} \equiv \sigma^M_{LK} = \frac{\beta L^{n-1} + \alpha}{n\beta L^{n-1} + \alpha} \tag{18}$$

since it is no longer dependent upon K. As a result, $\lim_{L \to L^+} \sigma = \frac{\beta L^{n-1} + \alpha}{n\beta L^{n-1} + \alpha}$ and $\lim_{L \to +\infty} \sigma = 1/n$. It is easy to check that σ can only diminish as L increases.

In the case of three factors, (14) and (15) allow us to obtain, for L and K,

$$\sigma^A_{KL} \equiv \sigma^A_{LK} = \frac{\alpha q K^{m-1}E^{q-1} + \beta q L^{n-1}E^{q-1} + \delta q L^{n-1}K^{m-1}}{mq\alpha K^{m-1}E^{q-1} + nq\beta L^{n-1}E^{q-1} + nm\delta L^{n-1}K^{m-1}} \tag{19}$$

$$\sigma^M_{LK} = \frac{\alpha q K^{m-1}E^{q-1} + \beta q L^{n-1}E^{q-1} + n\delta L^{n-1}K^{m-1}}{mq\alpha K^{m-1}E^{q-1} + nq\beta L^{n-1}E^{q-1} + nm\delta L^{n-1}K^{m-1}} \tag{20}$$

$$\sigma^M_{KL} = \frac{\alpha q K^{m-1}E^{q-1} + \beta q L^{n-1}E^{q-1} + m\delta L^{n-1}K^{m-1}}{mq\alpha K^{m-1}E^{q-1} + nq\beta L^{n-1}E^{q-1} + nm\delta L^{n-1}K^{m-1}} \tag{21}$$

First we will point out analogies among the σ:

a) all three are determined by variable quantities of the three factors and by all parameters of the function but γ;

b) all three fractions have the same denominators. They are the part of the bordered Hessian Δ that remains after $\alpha\beta\delta Y^4$ has been assembled into a common factor, a term which cancels out with other parts of the formulae. The denominators of the various σ (three in the case of Allen and six for Morishima) are always the same. The differences lie in the numerators of the fractions and, specifically, in the third addend. In (19), $\delta L^{n-1}K^{m-1}$ is multiplied, like the first and second, by q; in (20) by n and in (21) by m. *Coeteris paribus*, the differences between the values of σ are determined by relations such as $q > n > m$, $m > n > q$ and so on.

The values of the limit elasticities in (19), (20) and (21), and the

remaining ones in Allen and Morishima (two and four respectively), are determined by the twelve $\gtrless$ relations existing between n, m and q. For reasons of space, in this paper we shall only consider $\sigma^A_{LK} \equiv \sigma^A_{KL}$, σ^M_{LK} and σ^M_{KL}, making use of (19), (20) and (21). Nevertheless, these enable us to make the following general statements:

a) in the CMC, by increasing the factors from two to three it is also theoretically possible that $\sigma^A_{ij} \equiv \sigma^A_{ji}$ assume > 1 values. In fact, while the limit elasticities of $\sigma^A_{LK} \equiv \sigma^A_{KL}$ for $L \to \check{L}^+$ (with K and $E \to +\infty$) and for $K \to \check{K}^+$ (with L and $E \to +\infty$) are $1/m$ and $1/n$ respectively (thus equal to the two factors function case), when $L \to +\infty$ (with $K \to \check{K}^+$, $E \to \check{E}^+$) the limit elasticity results as the product of two fractions: $[q(n + q - 2)]/[n(2nq - n - q)]$ and q/n. The former is always < 1, whereas the latter may be $\gtrless 1$; therefore the product may result $\gtrless 1$. The same outcome may arise, *mutatis mutandis*, when $K \to +\infty$ (with $L \to \check{L}^+$, $E \to \check{E}^+$) and for the corresponding formulae of σ^A_{LE} and σ^A_{KE}. This is a fairly interesting result that is worth pointing out;

b) the value of parameter q, pertaining to the "third excluded" factor E, influences not only the value of σ^A_{LK} and of its limits, but also its increasing or decreasing movements between upper and lower bounds;

c) the picture changes with Morishima's σ. Both σ^M_{LK} and σ^M_{KL} have the same limit elasticities as the Allen's ones for $L \to \check{L}^+$ and $K \to \check{K}^+$, viz. $1/m$ and $1/n$. The limit elasticities of σ^M_{LK} for $L \to +\infty$ and $K \to +\infty$ are $[q(m + n) - q - n]/[n(2mq - m - q)]$ (a ratio always < 1) and $1/n$ respectively. The corresponding ones for σ^M_{KL} are $1/n$ and $[q(m + n) - q - m]/[m(2nq - n - q]$, both always < 1. Similar formulae and "symmetries" are valid, obviously *mutatis mutandis*, for other σ^M_{ij} and σ^M_{ji};

d) the parameter of the "third factor", q, still influences σ^M_{KL} and σ^M_{LK}, but not their movements between the upper and the lower bounds, in which only n and m, viz. the specifc parameters of L and K, are involved.

Matters are simplified if one of the factors appears in "Cobb-Douglas" form. Let us use the CMCE variant (E in "Cobb-Douglas" form) and consider σ_{LE}. The following emerges:

$$\sigma^A_{LE} \equiv \sigma^A_{EL} = \frac{m\alpha K^{m-1} + m\beta L^{n-1} + m\delta L^{n-1}K^{m-1}}{m\alpha K^{m-1} + n\beta L^{n-1} + nm\delta L^{n-1}K^{m-1}} \tag{22}$$

$$\sigma^M_{EL} = \sigma^M_{LE} = \frac{m\alpha K^{m-1} + n\beta L^{n-1} + m\delta L^{n-1}K^{m-1}}{m\alpha K^{m-1} + n\beta L^{n-1} + nm\delta L^{n-1}K^{m-1}} \tag{23}$$

The analogies between σ^A and σ^M, already verified for the complete CMC, are repeated here *mutatis mutandis*:

a) the Cobb-Douglas factor, E, disappears from *all* nine formulae of the σ and not just from $\sigma^A_{LE} \equiv \sigma^A_{EL}$. σ^M_{LE} and σ^M_{EL}; only its parameter δ – now dimensionless – remains, and only the movements of L and K are involved;

b) the denominators of the fractions expressing the nine σ are still equal, because they are the remainder of the specific Δ;

c) $\sigma^A_{LE} \equiv \sigma^A_{EL}$ differs from σ^M, but it may be seen that $\sigma^M_{LE} = \sigma^M_{EL}$. The (unexpected) result is therefore that, in a three-factor inhomogeneous function, the two σ^M between a Cobb-Douglas factor and non-Cobb-Douglas factor are identical. The same is obviously valid for $\sigma^M_{EK} \equiv \sigma^M_{KE}$.

The calculus of the limits shows that

$$\lim_{\substack{L\to L^+\\E\to+\infty\\K\to+\infty}} \sigma^A_{LE} = \frac{\alpha + \delta \check{L}^{n-1}}{\alpha + \delta n \check{L}^{n-1}} < 1; \quad \lim_{\substack{L\to+\infty\\E\to 0^+\\K\to K^+}} \sigma^A_{LE} = \frac{m}{n}\frac{\beta + \delta \check{K}^{m-1}}{\beta + \delta m \check{K}^{m-1}} \gtrless 1$$

$$\lim_{\substack{L\to L^+\\E\to+\infty\\K\to+\infty}} \sigma^M_{LE} = \frac{\alpha + \delta \check{L}^{n-1}}{\alpha + \delta n \check{L}^{n-1}} < 1; \quad \lim_{\substack{L\to+\infty\\E\to 0^+\\K\to K^+}} \sigma^M_{LE} = \frac{n\beta + \delta m \check{K}^{m-1}}{n(\beta + \delta m \check{K}^{m-1})} < 1$$

First of all it should be remembered that σ remains a pure, dimensionless numer, because α and β have dimension L and K respectively. The value of σ^A_{KL}, that is between the two non Cobb-Douglas factors, is always < 1 as σ^M_{KL} and σ^M_{LK}, even if the three values and formulae are obviously different.

It can be shown that in the production function with two Cobb-Douglas factors the matters become simpler.

4. *The Cost Function*

If, in the complete CMC, we substitute variables K and E, for example, with their value given by the expansion path (13), we obtain the equation

$$Y = \gamma \exp\left[\frac{\alpha}{1-n} L^{1-n} + \frac{\beta}{1-m}\left(\frac{\beta p_L}{\alpha p_K}\right)^{(1-m)/m} L^{\frac{n(1-m)}{m}} + \right.$$

$$\left. + \frac{\delta}{1-q}\left(\frac{\delta p_L}{\alpha p_E}\right)^{(1-q)/q} L^{\frac{n(1-q)}{q}}\right] \qquad (24)$$

the exponent of which is an invariably negative function of L. It emerges that $\lim_{L\to 0^+} Y(L) = 0$, $\lim_{L\to+\infty} Y(L) = \gamma$, $Y'(L) > 0$ always.

Function (24) is monotonic, determined for $L > 0$, with values in the interval $(0-\gamma)$ and strictly increasing; consequently, it is *invertible*. Since

$$\lim_{L\to 0} Y'(L) = 0 \qquad \text{and} \qquad \lim_{L\to+\infty} Y'(L) = 0$$

function $Y(L)$ is increasing and concave towards the top up to a certain value of L, after which it becomes concave towards the bottom, tending to the asymptote γ. Its inverse function $L(Y)$ is defined in $(0-\gamma)$, with values in the set $L > 0$, and its pattern, which can be obtained from that of $Y(L)$, will be symmetric to the bisecting line of quadrant I on which this function may be plotted.

To obtain function $C(Y)$, implicit in (1), a standard procedure is followed: if, in (11), which defines the generic cost function, value L is replaced by $L(Y)$ and K and E by those emerging from the expansion path (13), we obtain

$$C = p_L L + p_K \left(\frac{\beta p_L}{\alpha p_K}\right)^{1/m} [L(Y)]^{n/m} + p_E \left(\frac{\delta p_L}{\alpha p_E}\right)^{1/q} [L(Y)]^{n/q} \qquad (25)$$

One drawback to (25) is that function $L(Y)$ can only be made explicit in special cases, i.e., when the values of n, m and q are interlinked by determinate relations. While this difficulty must be made plain, it should immediately be added that, with the aid of a normal PC, the cost function may be calculated and plotted to the desired degree of approximation, given γ, α, β and δ and the factor prices, for all triplets of values n, m and q [16].

The simplest special case is the one in which $n = m = q$, for which we obtain

$$C(Y) = \left\{\frac{1}{n-1}\left[\alpha + \beta\left(\frac{\alpha p_L}{\beta p_K}\right)^{1/(n-1)} + \delta\left(\frac{\delta p_L}{\alpha p_E}\right)^{1/(n-1)}\right]\right\}^{1/(n-1)} \cdot$$

$$\cdot \left[p_L + p_K\left(\frac{\beta p_L}{\alpha p_K}\right)^{1/n} + p_E\left(\frac{\delta p_L}{\alpha p_E}\right)^{1/n}\right] \cdot \left[-\ln\left(\frac{Y}{\gamma}\right)\right]^{1/(n-1)} \qquad (26)$$

[16] Given the parameters of the CMC function and p_L, p_K and p_E, with increasing values of L, the corresponding values of K and E may be calculated by (13) and Y by (1). $C(Y)$ is obtained simply be means of (11). $C(Y)/Y$ may then be calculated. Through a series of endeavours, value $C(Y)/Y = \min$ may be determined to the desired degree of accuracy.

It could be shown that the level of Y, called Y_F, corresponding to the inflection point of (26), that is to $C'(Y) = \min$, is given by

$$Y_F = \gamma \exp\left(\frac{n}{1-n}\right) \tag{27}$$

while the level of Y, called $Y_{\bar{c}}$, corresponding to the stationary value of (26), that is to $C(Y)/Y = \min = \bar{c} = C'(Y)$, comes out to be

$$Y_{\bar{c}} = \gamma \exp\left(\frac{1}{1-n}\right) \tag{28}$$

By substituting (27) and (28) into (26), the values of C_F and $C_{\bar{c}}$ can be obtained [17]. $Y_{\bar{c}}$ is always greater than Y_F and both depend on γ and n only. It should be pointed out at once, however, that these two dependencies *are only valid when* $n = m = q$. In all other cases, including those in which function $L(Y)$ can be made explicit, at the parity of p_L, p_K and p_E, also the parameters α, β and δ influence the values of Y_F and Y_E. In order to obtain slightly more accurate notions about the behaviour of functions $C(Y)$ and $C(Y)/Y$, a large number of simulations were carried out, with various combinations of parameter values [18]. The results of these simulations may be summed up as follows:

a) *coeteris paribus*, an increase in γ – for instance, if it is doubled – produces the same increase in Y_F and $Y_{\bar{c}}$ and vice-versa. Likewise, function $C(Y)$ also shifts forward (or backwards) along the Y axis. For example, the value of $C(Y)$ with $\gamma = 50{,}000$ and $Y = 2{,}000$ is the same as that with $\gamma = 100{,}000$ and $Y = 4{,}000$;

b) *coeteris paribus*, increasing values of n, m and q determine *increasing* optimal sizes ($C(Y)/Y = \min$) of the firm [19];

c) $C(Y)$ and $C(Y)/Y$ rise when α, β and δ increase. When these parameters have high values, the existence of profit may only be obtained with equally high prices of Y [20];

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[17] See CANTARELLI et al. (1990, p. 67-74) for a detailed analysis of (26), (27) and (28) for the two factor function, but also valid for three or more.

[18] See CANTARELLI et al. (1990, pp. 75 and 125).

[19] A mechanism which accurately defines why and when large or increasing sizes of firms are possible and appropriate may also operate in a function with diminishing ε_Y.

[20] Unlike n, m and q, high values of α, β and δ make large sizes with uncertain profitability compulsory. Points b) and c) bear up our statements about *MP* in Section 2.

d) a situation in which n, m and q are equal or only slightly different is – *coeteris paribus* – more efficient than one in which the values of n, m and q are very different;
e) of course, the factors' price levels play their own part. For example, a high q value and a low δ value can be neutralised and overridden by a high level of p_E and vice-versa;
f) except in special cases (theoretic rather than practical ones), functions $C(Y)$ and $C(Y)/Y$ are U-shaped, and the shape of the U, which is more or less open and flat, is determined by the combination of parameters [21].

If one factor appears in "Cobb-Douglas" form, as in the CMCE variant, function $L(Y)$ can be made explicit with $n = m$. This is never possible in variants like CMCKE. However, this hurdle can be at least partially avoided in order to determine the "position" $C(Y)/Y = \bar{c} = \min = C'(Y)$ directly, with function parameters and factor prices. This "position" may be obtained by making use of the existing relationship between the total *passus* coefficient ε_Y and the elasticity of the cost function ε_c. In a "regular" inhomogeneous function, i.e., one in which ε_Y changes inversely to Y, as occurs in the CMC model, we always have $\varepsilon_Y = 1/\varepsilon_c$ (Ferguson, 1971), so that when $\varepsilon_Y = 1$, then ε_c must also be equal to 1.

In the CMCE variant, *and only with* $n = m$, when $\varepsilon_Y = 1$, we have

$$\varepsilon_Y = \alpha L^{1-n} + \beta K^{1-n} + \delta = 1 \tag{29}$$

If we substitute the value of K as a function of L, obtainable from the expansion path in (13), we obtain

$$L^{1-n}\left[\alpha + \beta\left(\frac{\beta p_L}{\alpha p_K}\right)^{\frac{1-n}{n}}\right] + \delta = 1 \tag{30}$$

The value of L which satisfies the condition expressed by (30) is one which enables us to obtain $\bar{Y}_{\bar{c}}$ and $\bar{c}$, and which we call $L_{\bar{c}}$. Therefore

$$\bar{L}_c = \left[\frac{\alpha + \beta\left(\frac{\beta p_L}{\alpha p_K}\right)^{\frac{1}{n-1}}}{1-\delta}\right]^{\frac{1}{n-1}} \tag{31}$$

[21] If, in the two-factor CMCK variant β were equal to 1, the mean cost would diminish towards a positive lower asymptote for $Y \to +\infty$; if $\beta > 1$, the average cost $\to 0$ for $Y \to +\infty$.

Putting in the R.H.S. of (31) $= a$, $(\delta p_L/\alpha p_E)^{\delta} = b$ and $(\beta p_L/\alpha p_K)^{1-n} = c$, then

$$Y_{\bar{c}} = \gamma b \exp\left[\frac{\alpha}{1-n} a^{\frac{1-n}{n-1}} + \frac{\beta}{1-n} c a^{n(1-n)}\right] a^{\delta n} \tag{32}$$

Replacing K and E by their values, given by the expansion path, the cost function (11) becomes

$$C_{\bar{c}} = p_L L + p_K \left(\frac{\beta p_L}{\alpha p_K}\right)^{1/n} L + p_L \frac{\delta}{\alpha} L^n$$

If we replace L with the R.H.S. of (31), we have

$$C_{\bar{c}} = p_L \left[a + \frac{\delta}{\alpha} a^n\right] + p_K \left(\frac{\beta p_L}{\alpha p_K}\right)^{1/n} a \tag{33}$$

If we divide (33) by (32) we obtain $\bar{c}$. If K is also expressed in "Cobb-Douglas" form (CMCKE variant), the equation corresponding to (29) is

$$\varepsilon_Y = \alpha L^{1-n} + \beta + \delta = 1 \tag{34}$$

from which we directly obtain

$$L_{\bar{c}} = \left(\frac{\alpha}{1-\beta-\delta}\right)^{\frac{1}{n-1}} \tag{35}$$

Using a similar double substitution in the cost function, we obtain

$$C_{\bar{c}} = p_L \left\{\left(\frac{\alpha}{1-\beta-\delta}\right)^{\frac{1}{n-1}} + \frac{\beta\delta}{\alpha}\left(\frac{\alpha}{1-\beta-\delta}\right)^{\frac{n}{n-1}}\right\} \tag{36}$$

The relation $\varepsilon_Y = 1/\varepsilon_c$ may therefore be used only when $r - 1$ factors can be expressed as a function of factor r. For this reason, it is useless in the complete CMC involving two factors or more, with $n \neq m \neq q$ or $n = m \neq q$ and so on. All the other points of functions $C(Y)$ and $C(Y)/Y$ will be obtained in the usual way.

Lastly, it may be of interest to point out that, with inhomogeneous

functions such as those in the CMC model, the $Y_{\bar{c}}$ "position" (for each variant) is the only one that ensures the product's exhaustion.

5. *A Few Econometric Exercises*

Before expounding the results of the econometric estimates, some observations about data, estimation procedures and methods are called for. First of all it should be pointed out that the CMC model was devised as an instrument for microeconomic analysis, although its application at macroeconomic level cannot – with due caution – be excluded *a priori*. As far as data are concerned, our preference is given to cross-section and panel data in comparison with time-series [22]. Such a preference stems from the fact that they provide a breakdown (or allow one to be made) of firms according to their dimension, which is measured in terms of product or added value, and in terms of the amount of production factors [23]. As far as we are concerned, this classification is of crucial importance because the chief objectives of our research include: a) checking whether there are structural differences between the production functions of small and large firms; b) verifying whether, in belonging to a given industry or branch of activity, dimension can be a more unifying structural factor than the type of goods produced [24].

Stage-wise and direct estimation procedures can both be applied to the CMC model. The first is carried out through the equation of the expansion path, in order to get estimates of n, m and q. Such a choice seems quite obvious and natural for a function "generated" by the expansion path itself. The implicit hypothesis of the long-run average cost minimization is, *a priori*, undoubtedly wiser and less demanding than other ones, such as short-run profit maximization and perfect competition, which are the basis of esti-
⁻tes of n, m and q carried out through marginal productivities or output
s equations. In all three cases factor prices are needed [25]. With two

[22] The ideal set of data should be obviously – besides panel data – a number of cross-sections at different times, including "nicely" scattered data concerning firms producing a homogeneous good or a clearly defined "basket" of goods.

[23] Partition does not necessarily mean splitting data into classes (which will rarely be numerous), obtaining the mean values and using them to estimate the model, although in practice this nearly always occurs.

[24] These phenomena may be encountered in the Italian manufacturing industry (see CANTARELLI et al., 1990).

[25] While it is relatively easy to find sufficiently reliable data for p_L and p_E, matters become more complicated for p_K, the "price" of capital (understood as both stock and flow of

factors (L and K, for instance) the (modified) equation of the expansion path can be used, viz.

$$\ln\left(\frac{K}{L}\right)_i = b_0 + b_1 \ln\left(\frac{p_L}{p_K}\right)_i + b_2 \ln L_i + u_i \tag{37}$$

where the subscript i denotes variables pertaining to the i-th unit ($i = 1, \ldots, N$), $b_0 = \frac{1}{m}\ln\left(\frac{\beta}{\alpha}\right)$, $b_1 = \frac{1}{m}$, $b_2 = \frac{n-m}{m}$ and u_i is a zero-mean random error term.

When there are three factors, it is necessary to get estimates by the simultaneous non-linear model

$$\begin{cases} \ln K_i = \frac{1}{m}\ln\frac{\beta}{\alpha} + \frac{1}{m}\ln\left(\frac{p_L}{p_K}\right)_i + \frac{n}{m}\ln L_i + u_{1,i} \\ \ln E_i = \frac{1}{q}\ln\frac{\delta}{\alpha} + \frac{1}{q}\ln\left(\frac{p_L}{p_E}\right)_i + \frac{n}{q}\ln L_i + u_{2,i} \end{cases} \tag{38}$$

where $u_{1,i}$ and $u_{2,i}$ are zero-mean random error terms [26]. Models like (38) are usually estimated using instrumental variables. It must be admitted that the choice (and availability) of instrumental variables is not always straightforward [27].

After having obtained the estimates of n and m and conducted tests on their significance against 0 and 1, the variables

$$L^* = \frac{1}{1-n}L^{1-n}; \quad K^* = \frac{1}{1-m}K^{1-m}; \quad E^* = \frac{1}{1-q}E^{1-q}$$

are calculated. Ordinary Least Squares (OLS) are then used to estimate the log-transformed relationship

$$\ln Y_i = \ln\gamma + \alpha L_i^* + \beta K_i^* + \delta E_i^* + u_i \tag{39}$$

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services). Experience has taught us that the rental price of capital calculated in the well-known way: (added value − total labour cost)/fixed capital, is a rather poor proxy.

[26] We could assume that the u_i's are normally and independently distributed with common variance.

[27] A possible and appropriate solution lies in using the lagged values of variables. In cross-sections, however, these data are rarely available.

In this way, estimates of γ, α, β and δ may be obtained.

If factor prices are unavailable, the direct linear or a non-linear estimation procedure can be applied [28]. The linear one is carried out through intensive computer work. Starting out from the triplet $n = m = q = 1$, i.e. from the "Cobb-Douglas" form, we proceed with a "step" of varying length in which we calculate the values of L^*, K^*, E^* and then estimate (39). The values of $\bar{R}^2$ and the empirical t-ratios will supply the indications needed to proceed, and tell us in which direction to go and when to stop. We are quite willing to acknowledge that the procedure is somewhat rough and crude. However, one also has to admit that it is no worse than Taylor's serial approximation procedure, and that it enables us to determine beyond doubt the highest $\bar{R}^2$ and the best t [29]. The procedure – linear or non-linear – is neutral with regard to the hypothesis of long-run average cost minimization or short-run profit maximization. Lastly, the fact that side relations based on the above mentioned theoretic hypotheses are not used may be regarded as an advantage, because a direct estimate "... posits only the production function, only one theory" as its basis [30].

The first exercise was performed on data originally used by F.R. Førsund and L. Hjalmarsson, concerning 28 general dairy plants in Sweden in the period 1964-1973. The output is expressed in tons of milk processed *per annum*; there are three factors of production: labour (hours worked *per annum*), aggregate capital (users' cost of, at 1964 prices) and energy cost [31].

Since factor prices were not available, we were unable to use the expansion path to get an estimate of n, m and q and to calculate the cost functions. The estimation was therefore conducted using the OLS direct method for the entire sample, with all the firms (ALL sample or "group"), and for six subsamples (called "groups"), i.e.: small and large firms in terms of labour (LS and LL), in terms of capital (KS and KL) and small and large firms in terms of labour and capital (LSKS and LLKL). The criterion for allocation to each group was based on the mean values $\bar{L}$ and $\bar{K}$ of the whole sample. The estimate for the first four groups was carried out chiefly

[28] In these "exercises" we used simple estimation methods to get a preliminary picture of the empirical performance of the CMC model, deferring the use of more suitable variance-components models and/or non-linear estimation procedures to a next paper.

[29] The regression with maximum $\bar{R}^2$ is rejected if it presents for at least one estimated parameter a value of the two-tail significance > 0.05. We should make it quite clear that, in the course of over 200 experiments, the surfaces of the solid $\bar{R}^2 = f(n, m)$ with $q = \text{const.}$, (or other similar ones), *never* presented more than one maximum.

[30] See BROWN (1968, p. 133).

[31] See FØRSUND and HJALMARSSON (1979).

TABLE 1

CLASSIFICATION OF THE FOUR BEST REGRESSIONS.
SWEDEN, GENERAL DAIRY PLANTS, 1964-1973

Group and number of observations		1st	2nd	3rd	4th
		n, m, q	n, m, q	n, m, q	n, m, q
		$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$
		name	name	name	name
ALL	280	1.2; 1.0; 1.0 .82688 CMCKE	1.3; 1.0; 1.0 .82679 CMCKE	1.2; 1.1; 1.0 .82670 CMCE	1.2; 1.2; 1.0 .82616 CMCE
LS	185	1.0; 1.0; 1.0 .71109 CD	1.2; 1.0; 1.0 .71059 CMCKE	1.3; 1.0; 1.0 .71027 CMCKE	1.5; 1.0; 1.0 .70947 CMCKE
LL	95	1.3; 1.0; 1.0 .82427 CMCKE	1.0; 1.2; 1.0 .82359 CMCLE	1.5; 1.0; 1.0 .82288 CMCKE	1.2; 1.2; 1.0 .82264 CMCE
KS	166	1.0; 1.0; 1.0 .70267 CD	1.2; 1.0; 1.0 .70165 CMCKE	1.3; 1.0; 1.0 .70053 CMCKE	1.2: 1.2; 1.0 .69960 CMCE
KL	114	1.0; 1.5; 1.0 .83441 CMCLE	1.0; 1.5; 1.2 .83377 CMCL	1.0; 1.3; 1.0 .83359 CMCLE	1.0; 1.3; 1.2 .83326 CMCL
LSKS	135	1.0; 1.0; 1.0 .67392 CD	1.2; 1.0; 1.0 .67284 CMCKE	1.3; 1.0; 1.0 .67215 CMCKE	1.2: 1.2; 1.0 .67088 CMCE
LLKL	64	1.8; 1.8; 1.7 .89685 CMC	1.7; 1.7; 1.7 .89593 CMC-CES	1.6; 1.8; 1.7 .89592 CMC	1.6; 1.6; 1.6 .89277 CMC-CES

to ascertain whether there were any situations that especially deserved to be studied in greater detail. In our opinion, however, the two subsamples LSKS and LLKL are the most interesting ones, since these are genuinely

different groups for which data are not overlapping. Since a time-series of cross-sections was involved, variable t (time) was added to the three factors.

In the boxes in Table 1, the four best regressions for each group appear along the rows from left to right, in decreasing order of $\bar{R}^2$. The top row of every box shows the values assumed by the n, m and q triplet; the $\bar{R}^2$ appear in the second row and the name in the third. This table allows

TABLE 2

ESTIMATED PARAMETERS OF CMC MODEL.
SWEDEN, GENERAL DAIRY PLANTS, 1964-1973

Group and number of observations	n / m / q	lnγ / γ	μ	α	β	δ	$\bar{R}^2$ / F / s
ALL 280	1.20	5.67962	.029488	3.50552	.35514	.24479	.826989
	1.00	(7.549)	(4.568)	(10.317)	(6.108)	(6.306)	334.403
	1.00	292.839					.24787
ALL 280	1.20	13.46384	.030400	3.56868	1.15788	.24603	.826704
	1.00	(14.725)	(4.739)	(10.620)	(6.066)	(6.339)	333.739
	1.00	703517.47					.24808
LSKS 135	1.00	−3.05152	.03123	.66443	.25585	.33402	.673923
	1.00	(3.285)	(3.301)	(5.950)	(2.474)	(4.998)	70.236
	1.00	.04728					.27633
LSKS 135	1.20	10.28248	.03159	4.01699	2.39275	.348	.670886
	1.20	(8.296)	(3.325)	(5.970)	(2.279)	(5.132)	69.288
	1.00	29563.21					.27761
LLKL 64	1.80	12.58599	.02568	1121.3323	15634.392	1779.8880	.896856
	1.80	(101.601)	(2.369)	(3.754)	(5.215)	(7.081)	137.949
	1.70	292433.04					.166889
LLKL 64	1.70	12.74148	.02538	404.5061	4372.985	1778.787	.895931
	1.70	(94.241)	(2.319)	(3.622)	(5.091)	(7.035)	136.591
	1.70	341631.5					.167636

us to make several remarks: a) the values of the $\bar{R}^2$ do not differ greatly (contrary to findings in many other experiments), while there are considerable differences among the groups; b) the highest $\bar{R}^2$ for the ALL group is not, as frequently occurs in a wide sample, the Cobb-Douglas, which lies in the fifth place. The same applies to groups LL, KL and LLKL; c) the Cobb-Douglas comes first in the groups of small firms, LS, KS and LSKS; d) the structural differences between small and large firms, which Table 1 has already highlighted, are "mediated" in the total sample. It may be observed that the Cobb-Douglas of the LLKL group has an $\bar{R}^2 = .8609$, against the values, all $> .89$, reported in Table 1.

Table 2 contains the parameters of two OLS regressions for each of the three groups ALL, LSKS and LLKL of the equation $Y = f(t, L, K, E, u)$. As Table 1 shows, the first is the best; the second was chosen from nos. 2, 3 and 4 in order to present an alternative version of the CMC model. The numbers in brackets beneath the parameters are the values of Student's empirical t. An examination of the parameters confirms the remarkable diversity between LSKS and LLKL firms. For LLKL firms, the value of the asymptote γ (two complete CMC functions are involved) creates no problems, being considerably higher than the maximum empirical value of Y. The estimated parameter δ of energy remains extremely significant; the rate of disembodied technological progress is greater in small firms than in large ones. In the Cobb-Douglas of LSKS firms, the sum $\alpha + \beta + \delta$ is notably higher than 1; this result, however, must take a relatively low $\bar{R}^2$ into account. In actual fact, an examination of empirical data for the group also reveals inefficient situations for several firms.

The second exercise was also carried out using data for individual firms; however, unlike the firms examined in the exercise, they do not produce a specific, homogeneous good but a "basket of goods" for each production sector. INSEE, France, provided data from a "fichier" on 441 firms, divided into 14 sectors, covering 1967 and 1979. Added value Y and an estimate – reevaluated and corrected by INSEE – of gross capital assets K (both in thousands of current FF) were available for each firm, together with the average number of workers employed per year, L. Our selection from the 14 sectors was based on two criteria: a) the homogeneity (or, at least, relative homogeneity) of the contents of the "basket" of goods produced, and b) the size of the sample. In our view, certain sectors – building materials and glass, production of minerals and metals, textiles and clothing and food and agricultural industries – are insufficiently homogeneous, while the sector including wood, furniture, printing and leather is not

TABLE 3

ESTIMATED PARAMETERS OF THE CMC MODEL. FRANCE, 1967 AND 1979

Sector and number of observations	n / m	$\log \gamma$ / γ	α	β	F / s	$\bar{R}^2$
Rubber and plastic 1967 25	1.20 / 1.00	7.61124 (3.641) 2020.782	1.69247 (3.179)	.43733 (3.364)	291.863 / .30114	.96037
Mechanical engineering 1967 80	1.40 / 1.00	9.42064 (13.297) 12340.47	9.19580 (11.667)	.21838 (3.863)	474.352 / .30091	.92298
Automobiles and land vehicles 1967 23	1.00 / 1.00	2.03792 (11.028) 7.674	.77189 (14.966)	.25409 (5.872)	1488.686 / .10947	.99266
Casting and metal processing 1967 36	1.20 / 1.00	6.72111 (4.478) 829.738	1.53448 (4.385)	.49333 (4.927)	420.564 / .27369	.95996
Electrical and electronic equipment 1967 55	1.30 / 1.10	15.88779 (39.346)	2.90425 (10.306)	.54538 (2.845)	447.827 / .34613	.94302
Rubber and plastic 1979 25	1.20 / 1.00	9.77798 (6.088) 17640.98	2.18765 (5.859)	.37468 (3.975)	416.818 / .22235	.97195
Mechanical engineering 1979 81	1.40 / 1.00	11.04289 (11.811) 62498.010	10.23006 (11.094)	.19014 (2.814)	440.541 / .30761	.91658
Automobiles and land vehicles 1979 23	1.00 / 1.00	3.47057 (7.399) 32.1550	.78247 (7.159)	.21691 (2.336)	369.633 / .20060	.97102
Casting and metal processing 1979 37	1.00 / 1.10	8.96411 (5.430) 7817.421	.76295 (7.390)	.90051 (2.887)	1000.117 / .18645	.98230
Electrical and electronics equipment 1979 52	1.00 / 1.10	8.16400 (6.921) 3512.207	.80347 (11.301)	.70338 (3.116)	1038.240 / .21234	.97600

homogeneous at all. Size was fixed at a minimum of 20 data. The best estimates, carried out on the chosen sectors using the direct estimation procedure are reported in Table 3. In two sectors, the choice of regression created a few

doubts, due to the significance of the estimated parameters; however, these were resolved [32]. Table 3 shows that only the automobile and land vehicle sector is unquestionably Cobb-Douglas for both years. In fact, the only remaining criterion of choice was the max $\bar{R}^2$, since many $(n; m)$ pairs around of $(n = 1; m = 1)$, with $\bar{R}^2$ lower than the max $\bar{R}^2$, albeit by very little, have fully significant estimated parameters. For the other four sectors, and with regard to both years, the complete CMC function and its variants are applicable. The annual unit cost of L may be calculated for this set of data, as may the rental price of capital (if one believes in its expressiveness). These values could be used to perform some interesting processing operations – for instance, we could calculate the cost functions (or simulate them, using estimates of the unit cost of capitals as p_K) – and to conduct sensitivity analyses, i.e. to check how the cost function varies as changes take place in the $(n; m)$ pairs that supplied $\bar{R}^2$ values coming very close to the highest one [33].

Although the CMC model should be regarded as a typical instrument of microeconomic analysis, we also tested its application at macroeconomic level and with a time series. This was done using the KLEM data set prepared by Berndt and Wood (1975) and published by Berndt (1991). Factor prices are also included in this set, so that the estimate may be tested in two stages by means of the expansion path; this was carried out with K and L, with added value as dependent variable. Since a time series is involved, we first tried adding time variable t to the variables assumed to be independent in (37) and then also added the lagged value $\ln (K/L)_{t-1}$, which provided us with two alternatives, (37a) and (37b). In both cases, the estimated parameters b_1 in (37) differed significantly from 0 and from 1, supplying $m = 2.94903$ and $m = 4.15737$. In (37a), the value of b_2 is "incorrect" because it produces an $n < 1$ value; in (37b), b_2 is not significant. The application of the expansion path may, however, be perceived and interpreted as a preliminary test to establish which type of variant is valid [34]. In the case in question, if the function is a CMCL (i.e., with $n = 1$) rather than a complete CMC, b_2 should not be significantly different from $b_1 - 1$. The test was successful in both cases, making the CMCL

[32] Given two regressions a and b with $\bar{R}^2_a > \bar{R}^2_b$, if the two-tail significance of an estimated parameter in a is < 0.01 and if all the estimated parameters of b are fully significant (at a 0.001 significance level), a is chosen; if the two-tail significance of a parameter in a is > 0.01, b is chosen.

[33] If we accept the hypothesis of short-run profit maximization, n and m may be calculated using the equations of MP or product shares.

[34] See CANTARELLI et al. (1990, p. 226).

variant the most suitable one for KLEM data. The two regressions of the expansion path, and the corresponding estimates of γ, α and β obtained by means of (39), are shown in Table 4. Of course, a direct estimate was also carried out for verification purposes. The maximum $\bar{R}^2$ obtained was the one with $n = 1$ and $m = 1.6$. The function's form is therefore confirmed, with a smaller m value. Two further conclusions may be drawn from the comparison: a) it is an established fact that with time series – and, we would add, the expansion path – we always obtain estimates of σ which

TABLE 4

ESTIMATED PARAMETERS OF THE CMC MODEL.
KLEM DATA, U.S.A., 1947-1971

EXPANSION PATH

Equation	const.	$\ln\left(\frac{p_L}{p_K}\right)$	$\ln L$	t	$\ln\left(\frac{L}{K}\right)_{t-1}$	F s $\bar{R}^2$
(37a)	1.12358 (1.529)	.33909 (3.959)	–.69478 (3.577)	.02297 (4.691)		58.148 .04168 .87720
(37b)	1.12363 (1.863)	.24053 (3.136)	–.58484 (3.537)	.01786 (3.537)	.26363 (2.215)	44.626 .03421 .88354

PRODUCTION FUNCTION

Equation	n m	$\log \gamma$ γ	α	β	F s	$\bar{R}^2$
(37a)	1.00 2.94903	2.16289 (4.227) .11499	1.73408 (14.619)	29.31846 (2.073)	641.152 .04429	.98159
(37b)	1.00 4.15737	–2.47411 (6.193) .08423	1.80083 (18.885)	499.44873 (1.948)	628.687 .04466	.98124

are lower than those obtainable from cross-sections; b) although it is true that the difference cannot be erased, it is equally true that, as Berndt wrote more than fifteen years ago, if data are processed wisely, while the two estimates will never become reconciled, they can be brought closer together and do not contradict each other [35].

6. *Conclusions*

We hope that the contents of the previous sections have adequately shown how the CMC model seeks to achieve the generality, flexibility and realism which, as mentioned in Section 1, are required for production functions. As far as we are concerned, the generality and flexibility made possible by this model are more important than the complete CMC function in itself. In theory and in reality there is no hard-and-fast rule whereby, in concrete terms, it must always be regarded as the best. There is no statutory law proclaiming that *all* factors of production may only have diminishing – or initially increasing and subsequently diminishing – *MP* and *AP* functions, because such hypotheses can only be accepted or rejected after the relevant econometric tests have been performed. Generality, flexibility and even realism are also apparent in the ease with which a fourth or fifth factor of production may be introduced. The property of separability possessed by all variants of the model ensures that these factors may be brought in, and at the same time enables them to be aggregated. If, for example, we introduce two kinds of labour (skilled technicians and unskilled workers) or two kinds of capital assets (structures and fixed installations on the one hand and machinery and equipment on the other) or similar distinctions, the *a priori* hypothesis that these factors can have different productivity laws is preferable to the one in which they are assumed to be similar. Needless to say, the same applies to elasticities of substitution. On the basis of the tests, these may be unitary constants or different from one, variable within more or less wide bounds and – where Allen's σ_{ij}^{A} elasticities are concerned – they may also become greater than 1. The latter possibility is undoubtedly the most important (and the least expected) outcome of our extension of the model to three factors. Whether this will remain pure theory or will actually be accomplished along determinate expansion paths (the "places" in which σ_{ij} takes on real practical relevance) will invariably be decided by empirical tests. In the context of the CMC model, the issue of complementarity

[35] See BERNDT (1976).

among factors may be considered in a different, more articulate, way than has traditionally been the case. The action of "efficient complementarity" in more or less short intervals of factor and product quantities, which is expressed by parameters n, m and q, may benefit from or be harmed by parameters α, β and δ.

The "precipitate" of the model's properties and characteristics is the cost function because these – and, hence, the parameters' values – determine, on a par with prices, the more or less open U shape of the average long-run cost function.

If the above statements are at least considered "reasonable", if not "rational", and if the econometric exercises – performed without dragging in the tanks, the missiles and the whole sophisticated arsenal of the most updated econometrics – have established the model's "right to citizenship", then it is fair to start thinking about our next target, i.e., about demand functions of factors and about using more efficient and advanced econometric procedures. In other words, it means that we have offered more than a textbook example (which would be a somewhat modest contribution even if, to the best of our knowledge, it has so far been lacking) of a production function with initially increasing and subsequently diminishing MP.

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UN NUOVO MODELLO DI FUNZIONE DELLA PRODUZIONE NON OMOGENEA MULTIFATTORI

Viene presentato un nuovo modello di funzione della produzione non omogenea a tre fattori, che ammette alternative specificazioni, guadagnando in tal modo flessibilità e realismo. Dopo le caratteristiche generali (*MP*, *AP*, isoquanti, MRTS, sentiero di espansione e coefficiente di passus), vengono esposte le due più rilevanti: le elasticità di sostituzione e la funzione duale di costo. Esse permettono di mettere in evidenza alcuni nuovi e interessanti risultati (come le relazioni tra le elasticità parziali di sostituzione di Allen e Morishima). I risultati di alcuni semplici test, che concludono l'articolo, dimostrano la convenienza a proseguire la ricerca teorica ed econometrica.

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ON THE FACTOR DEMAND THEORY OF LABOUR-MANAGED AND PROFIT-MAXIMIZING FIRMS IN INDUSTRY EQUILIBRIUM. AN APPLICATION OF LE CHÂTELIER PRINCIPLE

by
HONG HWANG *, YAN-SHU LIN **, and CHAO-CHENG MAI ***

1. *Introduction*

In recent years, the behavior of a labour-managed firm which is based on the assumption that collective workers will seek to maximize profit per worker has attracted economists' attention. The pioneering work of Ward (1958) compares the policy of such a firm with that of a profit-maximizer under perfect competition and shows in particular that a labour-managed (henceforth LM) firm produces less than a profit-maximizing (henceforth PM) firm which has the same production technology. The analysis is further carried out by Domar (1966), Vanek (1970), Maurice and Ferguson (1972), Meade (1972, 1974), Furubotn (1976), Berman (1977), Gal-Or et al. (1980), Hey (1981), Hill and Waterson (1983), Haruna (1985), Neary (1988), Kahana (1989), Mai and Hwang (1989), Wolfstetter (1990), Oku-guchi (1991) and others. Among others, as one should expect, the law of demand for industry factor behaviors under an LM organization differs significantly from the well-established results in a competitive capitalistic system. Unfortunately, to our knowledge, the analysis in this direction has

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been largely ignored. The analysis of cooperative enterprises along this line has a practical relevance. This kind of operation system can be found in the eastern world such as Yugoslavia (Ward, 1958), USSR (Domar, 1966) and China (Liu and Wei, 1987). It also resembles the operations of some modern enterprises in the western world. For example, many hospitals and law firms operate in such a way as to maximize the income per member of the physician or lawyer staff. And the behavior of derived demand for physicians or lawyers becomes an important issue.

The short- and long-run input behaviors of a competitive capitalistic economy has been studied by a number of economists, notably Ferguson and Saving (1969) (henceforth F-S), Silberberg (1974), Portes (1971) and Heiner (1982). In particular, F-S have used a production function technique to demonstrate that the short- and long-run derived input demand functions have the substitution and expenditure effects that are analogous to the substitution and income effects in consumer demand theory. Instead of working with a production function, Portes (1971) employs the cost function which is dual to the particular production function to re-examine the results of F-S. Indeed, this dual approach allows the comparative static properties to be derived in an effortless way. However, this approach seems not suitable for LM firms as they are not cost-minimizers at any level of output. For a given output, an LM firm's input mix will be different from that chosen by a PM firm.

The purpose of this paper is to apply Le Châtelier Principle to characterize the short- and long-run industry level factor-demand implications of an LM firm and to show how these implications related to the traditional theory of a PM firm. It is also shown that Le Châtelier Principle lends itself to an easy identification of the substitution and income or expenditure effects as well as the short- and long-run effects and thereby facilitating the comparison of the qualitative content of the LM firm's behavior with that of its PM twin.

This paper is organized as follows. Section II introduces Le Châtelier Principle and restates the F-S results. Section III examines the short- and long-run responses of input demand to a change in factor price in an LM industry and compares these with that in a PM industry. Some concluding remarks are provided in the final section.

II. *A Revisit of F-S Results – An Application of Le Châtelier Principle*

Consider a firm in a perfectly competitive industry. The production

function of the firm is given by $q = f(x_1, x_2, ..., x_n)$, where q is the rate of physical output and x_i is the rate of use of the i-th input. Over the relevant range all marginal products are positive $\left(\text{i.e., } \frac{\partial f}{\partial x_i} \equiv f_i > 0, \right.$ $\left. i = 1, 2, ..., n\right)$ and monotonically decreasing (i.e., $f_{ii} < 0$ for all i). We denote the price of the i-th input by P_i and product price by P. Then the total cost is given by $C = \sum_{i=1}^{n} P_i x_i + T$, where T is amortized sunk cost. Assume that the industry is perfectly competitive in all factor markets and that factor supplies are perfectly elastic. Thus, P_i is parametrically given to the firm. In addition, one of the characteristics which distinguishes the short-run equilibrium from the long-run equilibrium is the adjustment of product price. Following F-S, we adopt the concept of long-run equilibrium in which the entry and exit of firms adjusts product price to equal the common minimum long-run average cost of all firms. With this in mind, we are in a position to examine the response of the firm to a change in factor price.

Without loss of generality, in what follows we will concentrate on the response of the firm to a change in the price of factor 1. Applying Le Châtelier Principle, the total effect of an increase in P_1 on its demand can be decomposed into two parts (See Silberberg, 1990, p. 126):

$$(1) \qquad \frac{dx_1}{dP_1} = \left[\frac{dx_1}{dP_1}\right]_{dq=0} + \left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP_1}\right]$$

where $\left[\frac{dx_1}{dP_1}\right]_{dq=0}$ (i.e., the change in P_1 with respect to x_1 when q is treated as *exogenous* variable) is referred to the substitution effect; while the product of $\left[\frac{dx_1}{dq}\right]_{dq=0}$ (i.e., the change in x_1 with respect to q when q is treated as *exogenous* variable) and $\left[\frac{dq}{dP_1}\right]$ (i.e., the change in q with respect to P_1) is the expenditure effect. Moreover, the effect of a change in P_1 on q in (1) can be further decomposed into the short- and long-run effects:

$$\frac{dq}{dP_1} = \left[\frac{dq}{dP_1}\right]_{dP=0} + \left[\frac{dq}{dP}\right]_{dP=0}\left[\frac{dP}{dP_1}\right] \tag{2}$$

The first-term in the right-hand side of (2) is the short-run effect as the product price is treated as constant while the second term is the long-run effect as the product price is allowed to adjust to equal the minimum long-run average cost. Substituting (2) into (1) yields:

$$\frac{dx_1}{dP_1}_{SR} = \left[\frac{dx_1}{dP_1}\right]_{dq=0} + \left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP_1}\right]_{dP=0} \tag{3}$$

$$\frac{dx_1}{dP_1}_{LR} = \frac{dx_1}{dP_1}\Big|_{SR} + \left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP}\right]_{dP=0}\left[\frac{dP}{dP_1}\right] \tag{4}$$

where subscripts *SR* and *LR* stand for short-run and long-run.

Equation (3) represents the *short-run* response of factor demand to the change in input price while equation (4) the *long-run* response.

Via (3) and (4), we are ready to restate the F-S results. For the PM firm, the objective is to maximize its profits:

$$\pi = Pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} P_i x_i - T \tag{5}$$

In the short run equilibrium where output price is constant, the first-order conditions for profit-maximization are given by:

$$Pf_i - P_i = 0 \qquad i = 1, 2, \ldots, n \tag{6}$$

Assume that a unique maximum exists. Total differentiation of (6) with respect to x_i, P_1 and P yields:

$$\begin{bmatrix} Pf_{11} & Pf_{12} & \cdots & Pf_{1n} \\ Pf_{21} & Pf_{22} & \cdots & Pf_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ Pf_{n1} & Pf_{n2} & \cdots & Pf_{nn} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dP_1 + \begin{bmatrix} -f_1 \\ -f_2 \\ \cdot \\ \cdot \\ -f_n \end{bmatrix} dP \tag{7}$$

Following F-S, we denote the determinants associated with the unconstrained and constrained quadratics by F^* and F respectively, and the corresponding cofactors by subscripts, where the rows and columns of F are numbered 0, 1, ..., n and of F^*, 1, 2, ..., n. Then it is easily shown that

$F_{00} = F^*$ and $F_{0i} = -\sum_{j=1}^{n} f_j F^*_{ij}$, $i = 1, 2, \ldots, n$. In addition, we simply assume that there are an even number of inputs. Hence, the second-order conditions require $F > 0$ and $F^* > 0$.

Solving (7) by Cramer's rule, we obtain:

$$\frac{dx_i}{dP_1}_{dP=0} = \frac{F^*_{1i}}{PF^*} \qquad i = 1, 2, \ldots, n \tag{8-1}$$

$$\frac{dx_k}{dP}_{dP=0} = \frac{-1}{PF^*} \sum_{i=1}^{n} f_i F^*_{ik} \qquad k = 1, 2, \ldots, n \tag{8-2}$$

Since $dq = \sum_{i=1}^{n} f_i dx_i$, we can use (8) to derive the effect of a change in P_1 or P on total output:

$$\left.\frac{dq}{dP_1}\right|_{dP=0} = \sum_{i=1}^{n} f_i \frac{F^*_{1i}}{PF^*} \tag{9-1}$$

$$\left.\frac{dq}{dP}\right|_{dP=0} = -\sum_{k=1}^{n} f_k \frac{\sum_{i=1}^{n} f_i F^*_{ik}}{PF^*} \tag{9-2}$$

In order to derive the expressions for both $\left[\frac{dx_1}{dP_1}\right]_{dq=0}$ and $\left[\frac{dx_1}{dq}\right]_{dq=0}$ in (3), we can formulate the following Lagrangian function for the constrained maximization problem:

$$L = \left[[Pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} P_i x_i - T\right] + \mu [\bar{q} - f(x_1, x_2, \ldots, x_n)] \tag{10}$$

The first-order conditions require:

$$L_{x_i} = -P_i - \mu f_i = 0 \qquad i = 1, 2, \ldots, n \tag{11-1}$$

$$L_{\mu} = \bar{q} - f(x_1, x_2, \ldots x_n) = 0 \tag{11-2}$$

It warrants mention that, by duality theory, the problem of maximizing the profit subject to a given output $\bar{q}$ as described by (10) is equivalent to that of minimizing total costs for a given output $\bar{q}$. Moreover, $\bar{q}$ is not

necessarily equal to the profit-maximized output as solved by (5). However, equations (5) and (10) will yield the same solution, if equations (6) and (11-1) are equal in the equilibrium, that is, $\mu = -P$.

Now, displace the equilibrium system (11) by a change in P_1 and q. The resulting system may be written as follows;

$$(12)\qquad \begin{bmatrix} 0 & -f_1 & -f_2 & \cdots & -f_n \\ -f_1 & -\mu f_{11} & -\mu f_{12} & \cdots & -\mu f_{1n} \\ -f_2 & -\mu f_{21} & -\mu f_{22} & \cdots & -\mu f_{2n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -f_n & -\mu f_{n1} & -\mu f_{n2} & \cdots & -\mu f_{nn} \end{bmatrix} \begin{bmatrix} d\mu \\ dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dP_1 + \begin{bmatrix} -1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dq$$

Using Cramer's rule, the comparative static effects for a given output are derivable as:

$$(13\text{-}1)\qquad \left(\frac{dx_1}{dP_1}\right)_{dq=0} = \frac{-F_{11}}{\mu F}$$

$$(13\text{-}2)\qquad \left(\frac{dx_1}{dq}\right)_{dq=0} = \frac{-\sum_{j=1}^{n} f_j F^*_{1j}}{F}$$

(13-1) is the substitution effect. Since $F > 0$, $F_{11} < 0$ and $\mu < 0$, we have a negative substitution effect. Using (13-2) and (9-1), the expenditure effect can be defined as:

$$(14)\qquad \left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP_1}\right]_{dP=0} = \left(\frac{-\sum_{j=1}^{n} f_j F^*_{1j}}{F}\right)\left(\frac{\sum_{i=1}^{n} f_i F^*_{1i}}{PF^*}\right)$$

Next, following F-S, the expenditure elasticity of factor 1 (η_{1c}) and the partial elasticity of substitution (σ_{11}) are defined as:

$$(15\text{-}1)\qquad \eta_{1c} \equiv \frac{dx_1}{dc}\frac{c}{x_1} = -\frac{\lambda c}{x_1 F}\sum_{j=1}^{n} f_j F^*_{1j}$$

$$(15\text{-}2)\qquad \sigma_{11} \equiv \frac{c}{x_1^2}\frac{F_{11}}{F}$$

where $\lambda = \frac{1}{P} = -\frac{1}{\mu}$.

Substituting (13-1) and (14) into (3) and using (15), we obtain:

$$\left(\frac{dx_1}{dP_1}\right)^{\pi}_{SR} = \frac{\lambda x_1^2}{c}\sigma_{11} - \frac{x_1^2 F}{\lambda c^2 F^*}(\eta_{1c})^2 \tag{16}$$

Note that whenever we wish to distinguish between variables referring to the profit maximizer and the profit-per-worker maximizer, we use superscripts π and S respectively. Clearly, equation (16) is identical to equation (2.8) in F-S. It should be noted that both terms on the right-hand side of (16) are necessarily negative, so the quantity demanded in the short-run varies inversely with price irrespective of the factor classification.

We now turn to the long-run response of factor demand to the change in input price. The long-run competitive equilibrium is characterized by the following conditions [1]:

$$Pf_i - P_i = 0 \qquad i = 1, 2, \ldots, n \tag{17-1}$$

$$\pi = Pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} P_i x_i - T = 0 \tag{17-2}$$

A total differentiation of (17) leads to the following matrix:.

$$\begin{bmatrix} f & 0 & 0 & \ldots & 0 \\ f_1 & Pf_{11} & Pf_{12} & \ldots & Pf_{1n} \\ f_2 & Pf_{21} & Pf_{22} & \ldots & Pf_{2n} \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ f_n & Pf_{n1} & Pf_{n2} & \ldots & Pf_{nn} \end{bmatrix} \begin{bmatrix} dP \\ dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} x_1 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dP_1 \tag{18}$$

Applying Cramer's rule, we have:

$$\frac{dP}{dP_1} = \frac{x_1}{f} \tag{19}$$

Using (19), (13-2) and (9-2), we find:

$$\left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP}\right]_{dP=0}\left[\frac{dP}{dP_1}\right] = \left(\frac{x_1^2 F}{\lambda c^2 F^*}\right)\eta_{1c} \tag{20}$$

[1] For the long-run interpretation of sunk costs and their implications for entry, see BAUMOL and WILLIG (1981).

Substituting (20) and (16) into (4) and rearranging yields:

$$\frac{dx_1}{dP_1}^{\pi}_{LR} = \frac{\lambda x_1^2}{c}\sigma_{11} + \frac{x_1^2 F}{\lambda c^2 F^*}(\eta_{1c} - \hat{\eta}_{1c}) \tag{21}$$

This is precisely the decomposition in F-S (2.10), or in Portes (13). Manifestly, Le Châtelier Principle gives readers a more intuitive grasp of what is going on.

III. *The Factor Demand Theory of Labour-Managed Firm* [2]

Without loss of generality, we designate the n-th factor as labour and the rest of factors as non-labour inputs. The LM firm's objective is to maximize profit per worker, i.e., its problem can be stated as:

$$S = \frac{\pi}{x_n} = \frac{Pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} P_i x_i - T}{x_n} \tag{22}$$

Performing the maximization, we obtain the following first-order conditions:

$$Pf_i - P_i = 0 \qquad i = 1, 2, \ldots, n-1 \tag{23-1}$$

$$Pf_n - P_n - S = 0 \tag{23-2}$$

From (23), we have $\frac{f_i}{f_n} = \frac{P_i}{P_n + S} < \frac{P_i}{P_n}$. Apparently, these conditions differ from the efficiency conditions in a capitalist economy. This implies that the LM firm is not a cost-minimizer and that labour in an LM economy may be underemployed.

Total differentiation of (23) with respect to x_i, P_1 and P leads to the following matrix:

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[2] As pointed out by Bonin and Fukuda (1986, p. 174). Profit-per-worker maximization with amortized sunk costs requires the production function be decreasing returns to scale at the optimum. Thus, a restricted set of production technologies for which equation (23) characterizes the optimal choice of inputs should be defined here.

$$(24)\qquad \begin{bmatrix} Pf_{11} & Pf_{12} & \dots & Pf_{1n} \\ Pf_{21} & Pf_{22} & \dots & Pf_{2n} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ Pf_{n1} & Pf_{n2} & \dots & Pf_{nn} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ -\frac{x}{x_r} \end{bmatrix} dP_1 + \begin{bmatrix} -f_1 \\ -f_2 \\ \cdot \\ \cdot \\ -f_n + \frac{f}{x_n} \end{bmatrix} dP$$

Using Cramer's rule, we obtain [3]:

$$(25\text{-}1)\qquad \frac{dx_i}{dP_1}_{dP=0} = \frac{F^*_{1i} - \frac{x_1}{x_n} F^*_{ni}}{PF^*} \qquad i = 1, 2, \dots, n-1$$

$$(25\text{-}2)\qquad \frac{dx_k}{dP}_{dP=0} = \frac{-\sum_{i=1}^{n-1} f_i F^*_{ik} - \left(f_n - \frac{f}{x_n}\right) F^*_{nk}}{PF^*} \qquad k = 1, 2, \dots, n-1$$

Again, using the relation $dq = \sum_{i=1}^{n} f_i dx_i$, we obtain from (25) that:

$$(26\text{-}1)\qquad \frac{dq}{dP_1}_{dP=0} = \sum_{i=1}^{n} f_i \frac{F^*_{1i} - \frac{x_1}{x_n} F^*_{ni}}{PF^*}$$

$$(26\text{-}2)\qquad \frac{dq}{dP}_{dP=0} = -\sum_{k=1}^{n} f_k \frac{\sum_{i=1}^{n-1} f_i F^*_{ik} + \left(f_n - \frac{f}{x_n}\right) F^*_{nk}}{PF^*}$$

With this information, we wish to derive the substitution effect by constructing the following Lagrangian function:

$$(27)\qquad L^s = \frac{1}{x_n}\left[Pf(x_1, x_2, \dots, x_n) - \sum_{i=1}^{n} P_i x_i - T\right] + \\ + \mu\left[\bar{q} - f(x_1, x_2, \dots, x_n)\right]$$

The first-order conditions are:

[3] In this paper, the comparative statics are limited to non-labour inputs. Interested readers are welcome to conduct the comparative statics on the labour input and output of the LM firm.

$$L^s_{x_i} = \frac{-P_i}{x_n} - \mu f_i = 0 \qquad i = 1, 2, \ldots, n-1 \tag{28-1}$$

$$L^s_{x_n} = \frac{-(P_n + s)}{x_n} - \mu f_n = 0 \tag{28-2}$$

$$L^s_{\mu} = \bar{q} - f(x_1, x_2, \ldots, x_n) = 0 \tag{28-3}$$

Totally differentiating (28) and writing the resulting system of equations in a matrix form yields:

$$\begin{bmatrix} 0 & -f_1 & -f_2 & \cdots & -f_n \\ -f_1 & -\mu f_{11} & -\mu f_{12} & \cdots & \frac{P_1}{x_n^2} - \mu f_{1n} \\ -f_2 & -\mu f_{21} & -\mu f_{22} & \cdots & \frac{P_2}{x_n^2} - \mu f_{2n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -f_n & -\mu f_{n1} & -\mu f_{n2} & \cdots & \frac{P_n + S}{x_n^2} - \mu f_{nn} \end{bmatrix} \begin{bmatrix} d\mu \\ dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \tag{29}$$

$$\begin{bmatrix} 0 \\ \frac{1}{x_n} \\ 0 \\ \cdot \\ -\frac{x_1}{x_n} \end{bmatrix} dP_1 + \begin{bmatrix} -1 \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix} dq$$

By Cramer's rule, we have:

$$\left(\frac{dx_1}{dP_1}\right)_{dq=0} = \frac{-F_{11} + \frac{x_1}{x_n} F_{n1}}{x_n \mu F} \tag{30-1}$$

$$\left(\frac{dx_1}{dq}\right)_{dq=0} = -\frac{\sum_{j=1}^{n} f_j F^*_{1j}}{F} \tag{30-2}$$

Substituting (30) and (26-1) into (3) and using (15), we obtain:

$$(31) \qquad \frac{dx_1}{dP_1}^{S}_{SR} = \left(\frac{\lambda x_1^2}{cx_n}\sigma_{11} - \frac{\lambda x_1 F_{n1}}{x_n^2 F}\right) - \frac{x_1^2 F}{\lambda c^2 F^*}\eta_{1c}(\eta_{1c} - \eta_{nc})$$

where η_{nc} is the expenditure elasticity of labour.

Several points about (31) are warranted here. Firstly, the first term on the right-hand side of (31) captures the substitution effect. Comparing (31) with (16), we see that the qualitative difference of the substitution effect between the LM and PM firms is caused by the term $\left(-\frac{\lambda x_1 F_{n1}}{x_n^2 F}\right)$. Since $F > 0$ and $F_{n1} > 0$, the sign of this term is negative. As a result, the substitution effect of the LM firm is negative, which is similar to the one for the PM firm. Secondly, the second term on the right-hand side of (31) is the expenditure effect that reflects the change in output and expenditure incident to profit-per-worker maximizing adjustments to a change in factor price. This effect is in sharp contrast to that for the PM firm. In fact, whether the LM firm increases or decreases its factor demand as a result of a higher price of factor 1 depends on the relative magnitudes of the expenditure elasticities of factor 1 and labour. Either if labour is an essential input in the production process, (where being essential is defined as having a higher expenditure elasticity than the other inputs i.e., $\eta_{nc} > \eta_{1c}$) or if factor 1 is inferior, then an increase in the price of factor 1 may result in an increase in the usage of the factor. Therefore, *similar to the income effect in consumer demand theory, the expenditure effect of an LM firm can also be perverse in the short-run*. This result runs contrary to the well-developed production theory for a PM firm. According to F₹S, the expenditure effect of a PM firm can *never* be perverse. Finally, an intuitive explanation of these results can be given if account is taken of the dual role of labour in the context of an LM firm. A higher price of factor 1 implies a rise in total expenditure which induces the PM firm to reduce the demand for factor 1. However, the situation for an LM firm is more complicated because a higher expenditure may imply a larger increase in labour input (if η_{nc} is high), which itself may lower the value of the objective function defined as the profit per worker. Therefore, the increase in total expenditure (attributed to a higher price of factor 1) may not provide sufficient motivation for a lower demand for factor 1.

We now turn to the long-run industry factor-demand responses. As in the case of a PM industry, in the long-run, firms will enter (exit) an LM industry if positive (negative) profit per worker is being earned by the existing firm. Such a process will lead to a zero-profit per worker equilib-

rium. Hence, the conditions for the long-run competitive equilibrium in an LM industry are:

(32-1) $$Pf_i - P_i = 0 \qquad i = 1, 2, \ldots, n-1$$

(32-2) $$Pf_n - P_n - S = 0$$

(32-3) $$S = \frac{1}{x_n}\left[Pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} P_i x_i - T\right] = 0$$

Total differentiation of (32) leads to the following matrix;

(33) $$\begin{bmatrix} \frac{f}{x_n} & 0 & 0 & \ldots & 0 \\ f_1 & Pf_{11} & Pf_{12} & \ldots & Pf_{1n} \\ f_2 & Pf_{21} & Pf_{22} & & Pf_{2n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ f_n & Pf_{n1} & Pf_{n2} & \ldots & Pf_{nn} \end{bmatrix} \begin{bmatrix} dP \\ dx \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} \frac{x_1}{x_n} \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dP_1$$

Solving (33) by Cramer's rule yields:

(34) $$\frac{dP}{dP_1} = \frac{x_1}{f}$$

Using (34), (30-2) and (26-2), we can evaluate:

(35) $$\left[\frac{dx_1}{dq}\right]_{dq=0}\left[\frac{dq}{dP}\right]_{dP=0}\left[\frac{dP}{dP_1}\right] = \frac{x_1^2 F}{\lambda c^2 F^*}\eta_{1c} + \frac{\eta_{1c}x_1^2}{\lambda c x_n}\frac{\sum_k f_k F^*_{nk}}{PF^*}$$

Substituting (35) and (31) into (4) yields:

(36) $$\frac{dx_1}{dP_1}^{S}_{LR} = \left(\frac{\lambda x_1^2}{cx_n}\sigma_{11} - \frac{\lambda x_1 F_{n1}}{x_n^2 F}\right) + \frac{x_1^2 F}{\lambda c^2 F^*}(\eta_{1c} - \lambda_{1c}^2)$$

Surprisingly, the long-run expenditure effect is qualitatively equivalent to the one obtained in a PM industry. The reason for this is due to the fact that zero-profit per worker condition in an LM industry is identical to a zero- profit condition in a PM industry. However, it is important to recognize that since the substitution effect in an LM industry is different from that in a PM industry, the long-run industry factor demand responses to a change

in factor price are still different in both industries. As one might expect, the long-run responses are greater in an LM industry than in a PM one.

IV. *Concluding Remarks*

The principle of Le Châtelier concerns the movement of the stationary configuration of a constrained system when parameters are changed, and how this movement is affected by the presence of additional constraints. In this paper, this principle is applied to decompose the total effect of an increase in factor price on factor demand into the substitution and expenditure effects as well as the short-run and long-run effects. Utilization of this principle is shown to allow not only for a more elegant decomposition than is possible by other methods, but also for a more succinct comparison of an LM firm's behavior *vis-a-vis* that of its capitalistic twin. In particular, some of the important results can be summarized as follows:

1. Although the magnitude of the substitution effect in an LM industry is different from that in a PM industry, this effect is necessarily negative irrespective of the types of industries. In other words, all constant-output factor demand functions are negatively sloped with respect to their own prices.

2. The short-run response of input demand to a change in input price in an LM industry is in sharp contrast to that obtained in a PM industry. More specifically, the expenditure effect in an LM industry may have *perverse* response if labour is an essential input in the production process, where being essential is defined as having a sufficiently high expenditure elasticity.

3. The long-run expenditure effect in an LM industry is the same as that in a PM industry. However, since the substitution effect is different in both industries, the long-run responses are also different in both industries.

Whether our results illuminate characteristics which might in fact help to differentiate between labour-managed and profit-maximizing firms in western market economies is an empirical question. They may be most useful, however, in interpreting the behavior of an enterprise for which *a priori* considerations seem conclusively to justify the relevance of the profit per worker-maximization hypothesis used in Eastern European countries for many years, and where the response of input demands to change in external parameters at the micro level is particularly important.

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SULLA TEORIA DELLA DOMANDA DEI FATTORI DA PARTE DELLE IMPRESE GESTITE DAL LAVORO E DELLE IMPRESE CHE MASSIMIZZANO I PROFITTI NELL'EQUILIBRIO INDUSTRIALE. UN'APPLICAZIONE DEL PRINCIPIO DI LE CHÂTELIER

Lo scopo di questo articolo è di applicare il principio di Le Châtelier per caratterizzare, nell'equilibrio industriale di breve e lungo periodo, le implicazioni della domanda dei fattori di un'impresa gestita dal lavoro e di confrontarle con quelle della teoria tradizionale di un'impresa che massimizza il profitto. In particolare si mostra che nel breve periodo la risposta della domanda di fattori produttivi ad un cambiamento del loro prezzo in un'industria gestita dal lavoro contrasta decisamente con quella di un'industria che massimizza i profitti.

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CREDIBILITY AND POPULISM IN THE MANAGEMENT OF A PUBLIC SOCIAL SECURITY SYSTEM

by
LUIGI BONATTI *

Introduction and Motivations

In the discussion concerning the implications of an unfunded social security system on saving behavior, different arguments have been proposed to explain why an individual's pension wealth, which is the real present actuarial value of the social security benefits to be received, may not be a perfect substitute for non-pension wealth (see Dolde & Tobin, 1983, pp. 61-4).

Some of these arguments deal with the insurance dimension of public retirement schemes: in the presence of adverse selection and risk-adverse or risk-loving individuals, a change in the coverage against longevity risks provided by a public pension plan cannot be fully offset by a change in the purchasing of private insurances. This effect cannot but be accentuated by the fact that most public retirement schemes have redistributive scopes and imply a loose connection between the payroll taxes paid by an individual and the actuarial value of the benefits for which she/he is eligible. Moreover, the importance of liquidity constraints is such that the fact that pension rights are illiquid and cannot be cashed in advance or pledged as loans collateral makes mandatory pension saving an imperfect substitute for other assets. Finally, one should recognize – even without espousing the extreme view of Barro (1978) – that households that have some concern for their descendants may not perceive the whole present value of their future

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benefits as "net" wealth: they may offset (at least partially) any change in their pension wealth by changing the voluntary transfers to their children, who will have to meet the future obligations imposed by the social security system.

A complementary argument to be emphasized in the present paper is that future retirees may not perceive the whole value of the future benefits promised by a public pension scheme as "true" wealth, since they may have solid reasons to believe that these promises are excessive and will not be entirely kept. Such perception – that the commitments which are made towards future retirees by the authorities running the system may not be honored – may lower the degree of substitutability between (promised) pension wealth and non-pension wealth.

This problem of credibility has been probably important in a country like Italy. In spite of unfavourable demographic trends that are more accentuated than in other industrialized countries, the Italian public pension scheme has been extremely generous [1], relying on the possibility for the social security system to have budget deficits. Just recently (Fall 1992), a major crisis of confidence on the long-term sustainability of the increasing government debt made inevitable for a very reluctant political personnel to revise some of its mechanisms, approaching the institutional features that characterize the public pension schemes of other industrialized countries.

Padoa Schioppa (1992) states two reasons that make the problem of credibility more serious for the social security system than for other areas in which the government operates: *i*) By going into debt a "pay-as-you-go" system never finances the accumulation of productive assets, and this makes more likely that the debt will become unsustainable; *ii*) the revision or the annulment of the obligations contracted towards current or future retirees, who do not have many possibilities to retaliate, costs less for the government

[1] For a comparison of the Italian public pension scheme with the systems of other European countries, see PADOA SCHIOPPA (1992). According to Padoa Schioppa, the perception that the commitments of the social security towards future retirees may not be entirely met can explain the systematic evidence showing that in Italy the degree of substitutability between (promised) pension wealth and non-pension wealth is relatively low. Indeed, BRUGIAVINI (1987) finds for Italy that an increase in anticipated social security benefits, i.e. in benefits promised on the basis of the current social security legislation, relative to permanent income do reduce households' non-pension wealth, "but on average for much less than Lira-for-Lira, the implied offset [evaluated at the mean value of the sample of wealth holders] being between a 0.098% and 0.125% reduction in private wealth for one additional Lira of pension wealth" (p. 175). These estimated displacement rates are lower than those found for the U.S. and Canada in similar studies. For different estimates of the Italian replacement rate, see ROSSI and VISCO (1992).

than the partial or total repudiation of public debt, whose holders can punish the government by making more expensive the issuance of new debt [2]. It should be added to this second point that the government can make less visible the substantial revision of its commitments towards the retirees by changing only the indexation mechanisms to be used for revaluating the monetary benefits for which a retiree is eligible. Shifting from a consumer price index to a measure of the average nominal wage or to nominal GNP – or from a full to a partial indexation – can produce in a few years dramatic differences in the real value of the benefits.

However, the main point to be stressed is that a relatively long period of time is necessary to verify whether promises and commitments concerning the social security system will be kept, and that it is very unlikely that the politicians who made these promises will be still around when this time arrives. This implies that the authority managing the system in a given period *i*) can hardly feel (and be perceived as) responsible for the promises made by a former generation of politicians; and *ii*) it cannot invest in reputation by making only credible promises, since the verification and the possible reward would take place in a future that is beyond its horizon.

The realistic behavioral assumption underlying the present paper is that the political authority managing social security care only for its popularity among the living (and voting!) generations, which in their turn (and this might be less realistic) are assumed to have no concern for their descendants. Moreover, another stylized fact which is taken for granted is that the households who are aware that the government's promises are not credible do not punish it electorally, as far as they know that acting upon the belief that these promises are not to be taken seriously they can skip the consequences of their government's populism [3].

Consequently, the politicians of this paper have an incentive to an-

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[2] On these grounds, this paper rules out the possibility that the government reneges on its debt in order to focus only on the possibility that the government does not keep its promises concerning social security benefits. It is obvious that in the real world the total or partial repudiation of public debt is a much less likely contingency than the revision of the promises towards the retirees, but it is not a zero-probability event. If the wealth holders perceive the existence of a positive correlation between these two risks, a larger pension wealth, which is an illiquid asset, might raise the risk premium on the public debt (see GIRALDI et al., 1991).

[3] According to ONOFRI (1992), the persuasion of being smarter than their politicians is the clue to understand the attitude of most Italians towards their rulers in the last decade. Being aware of the unsustainability and myopia of the policies followed by their government, most Italians have stuck to the game in order to capture any possible personal gain from the situation, in the hope that when sooner or later there will be accounts to render they will be able – on a personal basis – to escape from any burden.

nounce "time-inconsistent" policies, i.e. policies that will be revised since they are suboptimal from the viewpoint of the period in which they have to be implemented, because there is some chance that the private agents are "naïve" instead of "ultrarational". The naïve households are those who believe that any announced policy which is "feasible" will be really brought about; where a feasible policy is a plan that, if correctly internalized by the private sector and implemented, is consistent with the solvency in the long run of the social security system (see Auerbach and Kotlikoff, 1987). On the other hand, the ultrarational agents are those who believe only in policies which are consistent with the constraints, the incentives and the objectives of the authorities which have to implement them. These smart households are aware that the politicians running the system in the future will not be willing to impose excessive burden on the young generations in order to keep the promises made in the present toward the current generations.

Therefore, the impact of an announced social security policy on private saving and capital formation depends on the fact whether the households are naïve or ultrarational. The depressing impact, which – consistently with the empirical evidence presented by Feldstein (1974) for the U.S. and by Rossi and Visco (1992) for Italy [4] – an unfunded social security system tends to have on private saving and capital formation, would be even more exacerbated by "populist" politicians in the absence of ultrarational agents. Indeed, these smart households save more to support their old-age consumption, since they anticipate that their retirement benefits will be lower than those promised by the government. Furthermore, in this economy without intergenerational altruism the Ricardian equivalence holds: being aware of the redistributive objectives of those managing the social security system, the active workers anticipate that there will be a reduction in their retirement benefits in response to a current increase in the share of the total outlays of the system in favour of the old which is financed by issuing debt.

In the tradition of Samuelson (1958, 1975) and Diamond (1965), an overlapping generations framework is used in this paper to analyze the effects of social security and government debt on saving and capital formation. Under the simplifying assumption of stationary population, Section 1 presents a model in which each generation has a two-period life cycle. Section 2 deals with a pay-as-you-go system, whose parameters are assumed

[4] Feldstein (1974) gave rise to a wide and lively debate in which contrasting empirical evidence has been presented to support competing theoretical standpoints on the impact of social security on private saving and capital formation. Among the many important contributions, see Barro (1978), Leimer and Lesnoy (1982), Feldstein (1982), Auerbach and Kotlikoff (1983).

to remain fixed forever. The dynamics of the economy around steady states with non-negative social security debt and interest rates is analyzed: given the parameters' values and the initial level of the capital stock, the stability properties of these steady states are shown to depend critically on the initial total transfers to the aged, both in the form of pension benefits and of interest plus principal payments on the outstanding debt of the social security system.

Section 3 studies the problem of credibility arising from the fact that those who manage social security care only about the welfare of the living generations and have effective control only on current policies, since now the parameters of the system can be revised at any time. We see that the implications on private saving and capital formation are different depending on whether the future retirees are ultrarational or naïve. Section 4 discusses some possible extensions of the analysis proposed in this paper.

1. *The Model*

The economy that we consider is composed of individuals, firms and a governmental agency operating inter-generational transfers.

Individuals. – Individuals live for two periods so that at any point in time the total population consists of two generations, the young and the old. For simplicity and without loss of generality, each cohort is assumed to have the same size: the number of individuals belonging to each generation is constant and is normalized to be one.

Individuals born at time t derive utility from consumption when young, C_{yt}, and consumption when old, C_{ot+1}, according to the additively separable lifetime utility function

$$u(C_{yt}) + \delta u(C_{ot+1}), \qquad 0 < \delta < 1 \tag{1}$$

where $u(C_{gt}) = \ln C_{gt}$, $\quad g = y, o, \quad t = 0, 1, 2, \ldots$.

Individuals work full time when young, supplying inelastically one unit of labor at a real wage W_t, and are retired when old. Their old-age resources are their savings plus income earned on these savings and net transfers from the government. Since individuals are not altruist, there are no voluntary intergenerational transfers (bequests, gifts): the elderly consume all their resources at the end of their last period of life.

The representative individual born at time t chooses consumption demands to maximize (1) subject to

$$C_{yt} + A_{t+1} \leq W_t + V_{yt} \qquad C_{yt} \geq 0 \tag{2}$$

and to

$$C_{ot+1} \leq (1 + r_{t+1}) A_{t+1} + V_{ot+1} \qquad C_{ot+1} \geq 0 \tag{3}$$

where A_{t+1} is net wealth accumulated in t, V_{yt} and V_{ot+1} are, respectively, net transfers from the governmental agency when young and when old, and r_{t+1} is the interest rate paid on savings held from period t to $t+1$.

Individuals form expectations consistently with the true process followed by the variables entering their budget constraints: in this deterministic context this assumption reduces to that of perfect foresight. Therefore, maximizing the lifetime utility in (1) subject to (2) and (3), we get the following solutions:

$$C_{yt} = \begin{cases} \dfrac{W_t + V_{yt} + V_{ot+1}(1+r_{t+1})^{-1}}{1+\delta} & \text{if } W_t + V_{yt} \geq -V_{ot+1}(1+r_{t+1})^{-1} \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

and

$$C_{ot+1} = \begin{cases} \dfrac{\delta[(W_t + V_{yt})(1+r_{t+1}) + V_{ot+1}]}{1+\delta} & \text{if } W_t + V_{yt} \geq \dfrac{-V_{ot+1}}{1+r_{t+1}} \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

Notice that if the net transfers to the old are strictly positive, an increase in the interest rate can never decrease total savings.

Firms. – Firms act competitively and their large number is normalized to be one. They produce a single product according to the Cobb-Douglas technology

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{6}$$

where Y_t is output, and K_t and L_t are, respectively, capital rent in period t and labor hired in t.

Profit maximization by the representative firm implies

$$(1-\alpha) K_t^{\alpha} L_t^{-\alpha} = W_t \tag{7}$$

and

$$\alpha L_t^{1-\alpha} K_t^{\alpha-1} = R_t \tag{8}$$

where R_t is the market rental rate on capital.

The governmental agency. Consistently with the Italian experience, the public agency running the system of mandatory intergenerational transfers does not hold productive assets and can go into debt to finance a current unbalance between receipts and outlays. Therefore, its budget constraint for period t is

$$B_{t+1} - V_{yt} = V_{ot} + B_t(1 + r_t) \tag{9}$$

where B_t is net borrowing from the governmental agency in period $t-1$ that must be returned with interest at time t. The choice of a sequence of transfers on the part of the government in period t is constrained by its intertemporal budget constraint

$$\frac{\sum_{s=0}^{\infty} (V_{ot+s} + V_{yt+s})}{\prod_{j=1}^{s} (1 + r_{t+j})} + B_t(1 + r_t) = 0 \tag{10}$$

Markets equilibrium. – The goods market equilibrium requires that

$$C_{yt} + C_{ot} + K_{t+1} = Y_t + (1 - \Phi) K_t \tag{11}$$

where for simplicity the depreciation rate of capital, Φ, is assumed to be one [5].

We have equilibrium in the labor market when the wage is such that firms desire to hire all the available workers:

$$L_t = 1 \tag{12}$$

Similarly, equilibrium in the market for capital services requires that the rental rate on capital is such that firms are inclined to use all the available capital stock:

$$K_t^d = K_t^s \tag{13}$$

Finally, for equilibrium in the capital market we need that

[5] This assumption does not imply any qualitative distortion and is realistic, since in this model the length of each period is equal to half of the life expectancy of an individual.

$$A_{t+1} = K_{t+1} + B_{t+1} \tag{14}$$

and that the following arbitrage condition be satisfied:

$$R_{t+1} = r_{t+1} + 1 \tag{15}$$

Notice that in this economy it is irrelevant for the elderly to hold their productive assets in the form of bonds or stocks: debt and equity are perfect substitutes.

2. *A Social Security System with Fixed Parameters*

Laissez-faire. – Suppose that the governmental agency does not play any role ($V_{yt} = V_{ot} = B_t = 0$ for all t). In this case, we can use (2), (4), (5), (7), (8), (12), (13), (14) and (15) to characterize the equilibrium path of the economy by the following nonlinear difference equation:

$$K_{t+1} = \frac{(1-\alpha)\,\delta K_t^{\alpha}}{(1+\delta)}, \qquad K_0 \text{ given} \tag{16}$$

Equation (16) reflects the fact that capital accumulation is fueled from the entire saving of the young generation, which saves a fixed proportion, $\delta/(1+\delta)$, of its income.

Given the dynamic behavior of the capital stock in (16), we have a unique steady state at which $K_{t+1} = K_t > 0$:

$$\lim_{t\to\infty} K_t = \bar{K} = \left[\frac{(1-\alpha)\,\delta}{1+\delta}\right]^{\frac{1}{(1-\alpha)}} \tag{17}$$

This steady state is locally stable: if we linearize (16) around $\bar{K}$, we obtain

$$K_{t+1} - \bar{K} = \alpha\,(K_t - \bar{K}) \tag{18}$$

To compare (17) to the "golden rule" steady state, K^*, at which the steady state level of total consumption is maximized, we solve

$$\max_{K} K^{\alpha} - K \tag{19}$$

from which we get

$$K^* = \alpha^{\frac{1}{(1-\alpha)}} \tag{20}$$

Notice that for $\delta > \alpha/(1-2\alpha)$ the steady state in (17) is dynamically inefficient ($\bar{K} > K^*$): the economy tends to overaccumulate capital.

A pay-as-you-go system. – Let us now introduce a pay-as-you-go social security system with fixed parameters allowing temporary unbalances between current recepits from payroll taxes and current benefit payments to the old:

$$V_{yt} = -\tau W_t \qquad 0 \leq \tau \leq 1 \tag{21a}$$

and

$$V_{ot+1} = \mu (1-\tau) W_t \qquad \mu \geq 0 \tag{21b}$$

where a proportion τ of their wage has to be paid by the young as social security contribution and a proportion μ of their after-tax wage when young is paid to the old as social security benefit.

To characterize an equilibrium path of the economy with the social security system abiding by the rules in (21), we use (2), (4), (5), (7), (8), (12), (13), (14) and (15) to write the budget constraint of the governmental agency in (9) as a second-order difference equation in K:

$$\tau (1-\alpha) K_t^{\alpha} + B_{t+1} = \mu (1-\tau) (1-\alpha) K_{t-1}^{\alpha} + B_t \alpha K_t^{\alpha-1} \tag{22}$$

where K_0, K_{-1} and B_0 are given, and

$$B_{t+1} = B(K_t, K_{t+1}) = \frac{(1-\tau)(1-\alpha)\delta K_t^{\alpha}}{1+\delta} - \frac{\mu (1-\tau)(1-\alpha) K_t^{\alpha} K_{t+1}^{\alpha-1}}{\alpha (1+\delta)} - K_{t+1}.$$

Steady states. – The steady states of this economy, at which $K_{t+1} = K_t = K_{t-1} > 0$, are:

$$\bar{K} = \begin{cases} \bar{K}^0 = \left[a + (a^2 - c)^{\frac{1}{2}} \right]^{\frac{1}{(\alpha-1)}} \\ \bar{K}^1 = \left[a - (a^2 - c)^{\frac{1}{2}} \right]^{\frac{1}{(\alpha-1)}} \end{cases} \tag{23}$$

and

$$c \equiv \frac{\mu (1-\tau)(1-\alpha) + \alpha (1+\delta)}{\delta \alpha^2 (1-\tau)(1-\alpha)}$$

where the parameters' values are assumed to be such that $a^2 \geq c$.

To compare the long-run behavior of this economy with social security to its behavior under laissez-faire, we realistically restrict our attention to dynamics around steady states at which the governmental agency is not a net lender. In the case in which $\delta < \alpha/(1-2\alpha)$, it turns out that only at $\bar{K} = \bar{K}^0$ with $\mu \leq \tau/(1-\tau)$ we have $B(.) \geq 0$. It is easy to check that in this case $\bar{K}^0 \leq \bar{K} < K^*$ and $\bar{K}^0 = \bar{K}$ if and only if $\mu = \tau = 0$: the steady-state capital stock is lower than in the absence of social security.

The well-known result that the introduction of an unfunded social security system depressed capital formation is due to the fact that the increased consumption of the initial elderly generation, which receives positive transfers from social security without having paid any contribution, will not be fully offset by a reduced consumption of the future generation (see Auerbach and Kotlikoff, 1987, pp. 146-9). In the present context, this intrinsic feature of an unfunded social security system can be exacerbated by the possibility for the public agency running the system to transfer additional resources to the elderly in the form of principal and interest payments on social security debt. Obviously, the typical effect on capital formation of an unfunded social security system can reduce or even remove the dynamic inefficiency of an economy that under laissez-faire would tend to overaccumulate (see Blanchard and Fischer, 1989, p. 113). This is true in the case in which $\delta > \alpha/(1-2\alpha)$: setting $\mu = \tau/(1-\tau)$ with $\tau < \dfrac{\delta(1-2\alpha)-\alpha}{(1-\alpha)(1+\delta)}$, we have $\bar{K}^0 = K^*$.

Dynamics around steady states. – The stability properties of $\bar{K}^0$ with $\mu \leq \tau/(1-\tau)$ if $\delta < \alpha/(1-2\alpha)$, and with $\mu = \tau/(1-\tau)$ and $\tau < \dfrac{\delta(1-2\alpha)-\alpha}{(1-\alpha)(1+\delta)}$ if $\delta > \alpha/(1-2\alpha)$, can be studied by linearizing (22) around $\bar{K}^0$, from which we get a second-order difference equation in $(K-\bar{K}^0)$. The characteristic equation associated with this difference equation has two roots, σ_1 and σ_2, which are such that $0 < \sigma_1 < 1$ and $\sigma_2 > 1$ [6].

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[6] By linearizing (22) around $\bar{K}^0$, we obtain $\tilde{K}^0_{t+1} = d\tilde{K}^0_t + e\tilde{K}^0_{t-1}$, $\tilde{K}^0_t \equiv K_t - \bar{K}^0$, $e \equiv \dfrac{-\alpha^2\delta(1-\tau)(1-\alpha)(\bar{K}^0)^{\alpha-1}}{\alpha(1+\delta)+\mu(1-\tau)(1-\alpha)} \times [\mu+\alpha(\bar{K}^0)^{\alpha-1}]$ and $d \equiv \dfrac{\alpha^2[\tau(1-\alpha)+\alpha+\delta](\bar{K}^0)^{\alpha-1} - \alpha\mu(1-\tau)(1-\alpha)+\alpha^2\delta(1-\tau)(1-\alpha)^2(\bar{K}^0)^{2(\alpha-1)}}{\alpha(1+\delta)+\mu(1-\tau)(1-\alpha)^2}$. The characteristic equation associated with this second-order difference equation is $G(\sigma) = \sigma^2 - d\sigma - e = 0$. It is easy to check that

This result implies that there exists only one equilibrium path converging to $\bar{K}^0$ (saddle-path stability), which must satisfy

$$(K_{t+1} - \bar{K}^0) = \sigma_1 (K_t - \bar{K}^0) \tag{24}$$

Given the capital stock outstanding in period 0 and the parameters of the social security system, the transfers to the initial elderly, both in the form of pension benefits and in the form of interest plus principal payments on social security's debt, must be such that the governmental agency needs borrowing exactly that amount which is consistent with an evolution of the capital stock along the path converging to $\bar{K}^0$. An excessive demand for funds in $t = 0$ by social security crowds out capital formation, in spite of the increased savings of the young in the initial period because of the higher interest rate, up to the point that the lower labor productivity and pre-tax-wage of the next period cannot support an adequate flow of saving. Over time, the governmental agency must increasingly finance its commitments by going into debt, thus leading to a further reduction of the capital stock that tends to zero. On the other hand, a too low deficit of social security in the initial period to be financed on the market produces an overaccumulation of capital, which implies a higher level of the tax base in subsequent periods and a dynamically inefficient stock of capital associated with a positive net debt for the private sector in the long run ($\bar{K} = \bar{K}^1 > K^*$ and $B(\bar{K}^1, \bar{K}^1) < 0$).

Conversely, the governmental agency may be able to specify a long-term policy in terms of combinations of τ and μ that – given K_0 and the initial transfers in favour of the old – is consistent with convergence to a steady state with non-negative capital and social security debt. It is intuitive that if the governmental agency inherits from the past a debt B_0, which is relatively high with respect to the productive potentialities of the economy (i.e. to K_0), its long-term policy cannot be too generous in order to be solvent in the long run: feasibility requires a social security tax rate τ which is relatively high with respect to μ (the ratio of benefits per retiree to after-tax labour income per capita). Under these circumstances, the economy can converge to a steady state exhibiting a relatively high social security debt-capital stock ratio.

Consider that a sequence of intergenerational transfers is feasible if the

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with $\mu \leq \tau/(1-\tau)$ if $\delta < \alpha/(1-2\alpha)$, and with $\mu = \tau/(1-\tau)$ and $\tau < \dfrac{\delta(1-2\alpha)-\alpha}{(1-\alpha)(1+\delta)}$ if $\delta > \alpha/(1-2\alpha)$, we have $G(0) > 0$, $G(1) < 0$ and $G(\infty) > 0$. This implies that $0 < \sigma_1 < 1$ and $\sigma_2 > 1$.

intertemporal budget constraint of the agency in (10) is satisfied along the path of the economy, which is realized in case the households believe that the governmental agency will follow exactly that course of actions. This constraint is certainly satisfied along the equilibrium path converging to $\bar{K}$.

3. *Credibility of Promises About Future Social Security Benefits*

In the discussion of the previous section, any feasible policy is implicitly considered credible: any announced sequence of intergenerational transfers, which is consistent with the solvency in the long run of the public agency if correctly internalized by the households and implemented, is perceived by the private sector as the "true" course of actions that the authorities will be following. This does not hold in general, in the sense that ultrarational agents can perceive that an announced policy – although feasible – can be inconsistent with the "true" objectives and incentives of those who decide on the intergenerational transfers in the present and of those who will decide on them in the future. In other words, feasibility is a necessary but not sufficient condition for the credibility of public policies. Thus, we can distinguish between the case in which the households are smart (or ultrarational), i.e. they act upon the hypothesis that only policies which are optimal from the viewpoint of those who have direct control on them will be really implemented, and the case in which the households are naïve, i.e. they believe that any announced policy which is feasible will be really implemented.

The optimizing behavior of the authorities. – To model more realistically the behavior of the authorities deciding on the intergenerational transfers, we assume that they care only about their popularity among those individuals who are currently alive, according to an utilitarian welfare function $w(.)$ that may weigh differently the young and the old:

$$w(C_{ot}, C_{yt}) = \pi u(C_{ot}) + (1-\pi)\, u(C_{yt}) \qquad 0 < \pi < 1 \tag{25}$$

Taking into account the optimizing behavior of the households, it is optimal for those managing the system in t to announce a pair of sequences $\{V^*_{os,t}\}_{s=t}^{\infty}$ and $\{V^*_{ys,t}\}_{s=t}^{\infty}$ and an associated sequence $\{B^*_{s+1,t}\}_{s=t}^{\infty}$ which solve

$$\text{Max } w(C_{ot}, C_{yt}) \tag{26}$$

subject to (2), to (4)-(15) and to (25), K_t and B_t given.

This plan of intergenerational transfers, which is optimal for the authority to announce in t no matter whether the households are naïve or ultrarational (see the Appendix), is such that

$$V^*_{ot,t} = \pi K_t^{\alpha} - A_t \alpha K_t^{\alpha-1} \tag{27a}$$

$$V^*_{yt,t} - B^*_{t+1,t} = -(\pi - \alpha) K_t^{\alpha} \tag{27b}$$

$$V^*_{ot+1,t} = K_{t+1}^{\alpha} - A_{t+1} \alpha K_{t+1}^{\alpha-1} \tag{27c}$$

$$V^*_{yt+1,t} = -W_{t+1} = -(1-\alpha) K_{t+1}^{\alpha} \tag{27d}$$

and

$$V^*_{ys,t} = V^*_{os,t} = 0 \text{ for all } s > t+1 \tag{27e}$$

This announced policy is a limiting case of a redistributive scheme aimed at favouring the currently living generations: without loss of generality, a minimum level of consumption for every future generations could be inserted in (26) as an additional constraint that a policy must satisfy for feasibility. Moreover, the inter-generational transfer in (27) can be operated through a social security system and/or through other institutional forms. It may be possible for the authority to formulate the entire redistributive scheme as an announced sequence of parameters' values of a pay-as-you-go system. In particular, those who manage the system in t may be able to specify a payroll tax rate $\tau^*_{t,t}$ for the young in t such that $\tau^*_{t,t} = -V^*_{yt,t} \times$ $\times [K_t^{\alpha}(1-\alpha)]^{-1}$ and a benefit-to-(net-of-tax) earnings rate $\mu^*_{t,t}$ for the old in t such that $\mu^*_{t,t} = V^*_{ot,t} [K_{t-1}^{\alpha}(1-\alpha)(1-\tau_{t-1})]^{-1}$, K_{t-1} and τ_{t-1} given, promising the young a similar treatment when they will retire ($\mu^*_{t,t} = \mu^*_{t+1,t}$) [7].

Naïve households. – If the households are naïve, the announced sequences of transfers in (27) are associated with

$$C^*_{ot} = \pi K_t^{\alpha} \tag{28a}$$

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[7] The authority in t may formulate the entire redistributive plan in (27) only as a specification of the parameters' values of a pay-as-you-go system, which promises to the generation that is young in t the same replacement rate enjoyed by the old, if $\mu^*_{t,t} = \mu^*_{t+1,t} =$ $= \dfrac{\pi K_t^{\alpha} - A_t \alpha K_t^{\alpha-1}}{(1-\tau_{t-1}) W_{t-1}} \geq 0$ and $\tau^*_{t,t} = 1 - \left[1 + \dfrac{\mu^*_{t,t}}{\alpha (K^*_{t+1})^{\alpha-1}}\right]^{-1} \times \dfrac{(1-\pi)(1+\delta)}{(1-\alpha)(1+\alpha\delta)} \geq 0.$ It is straightforward that the plan in (27) implies $\tau^*_{t+1,t} = 1$ and $\tau^*_{s,t} = \mu^*_{s,t} = 0$ for all $s > t+1$.

$$C_{yt}^{*} = \frac{(1-\pi)\, K_t^{\alpha}}{(1+\alpha\delta)} \tag{28b}$$

and

$$K_{t+1}^{*} = \frac{(1-\pi)\, \alpha\delta K_t^{\alpha}}{(1+\alpha\delta)} \tag{28c}$$

Notice that the consumption level of the young and the capital formation in t do not depend on the proportion of the total transfers to the old which is financed by borrowing instead of by taxing the young. In this economy without intergenerational altruism, the Ricardian equivalence holds because the redistributive policy announced by the authority implies that any change in the private assets held by the households who are currently young is fully offset by a change in the current value of the promised social security benefits.

As period $t+1$ arrives, it is optimal for those who manage the system to deviate from the policy announced in t. In fact, solving the problem in (26) for period $t+1$, we have

$$V_{ot+1,t+1}^{*} = \pi K_{t+1}^{\alpha} - A_{t+1}\, \alpha K_{t+1}^{\alpha-1} \tag{29a}$$

$$V_{yt+1,t+1}^{*} - B_{t+2,t+1}^{*} = -(\pi-\alpha)\, K_{t+1}^{\alpha} \tag{29b}$$

$$V_{ot+2,t+1}^{*} = K_{t+2}^{\alpha} - A_{t+2}\, \alpha K_{t+2}^{\alpha-1} \tag{29c}$$

$$V_{yt+2,t+1}^{*} = -\overline{w}_{t+2} = -(1-\alpha)\, K_{t+2}^{\alpha} \tag{29d}$$

and

$$V_{ys,t+1}^{*} = V_{os,t+1}^{*} = 0 \text{ for all } s > t+2 \tag{29e}$$

It is apparent by comparing (27) and (29) that $V_{ot+1,t}^{*} > V_{ot+1,t+1}^{*}$: the promise that the authorities made in t was not credible. On the basis of the true policy that is implemented in $t+1$, the generation which was young in t and believed in that promise would have preferred to save more in its youth.

In case every young generation is systematically deceived by the authorities, the capital stock evolves according to the difference equation in (28c). In this economy with systematic deception of the young, the steady state level of capital is

$$\bar{K}^{*} = \left[\frac{(1-\pi)\, \alpha\delta}{1+\alpha\delta}\right]^{\frac{1}{(1-\alpha)}} \tag{30}$$

Ultrarational households. – The possibility that the households can be deceived is inconsistent with the assumption of perfect foresight: in each t the ultrarational agents perceive that – no matter what plan is announced – the sequence of transfers which will be really implemented is $\{V_{ot,s}^{*}\}_{s=t}^{\infty}$, $\{V_{yt,s}^{*}\}_{s=t}^{\infty}$ and $\{B_{s+1,s}^{*}\}_{s=t}^{\infty}$ (see the Appendix). The optimal redistributive policy that is actually implemented in each period does not change if the authorities are aware of facing ultrarational instead of naïve households. The perfect-foresight equilibrium path, which is associated with the presence of ultrarational agents, is characterized by

$$C_{ot}^{**} = \pi K_t^{\alpha} \tag{31a}$$

$$C_{yt}^{**} = \frac{(1-\pi)\,\pi K_t^{\alpha}}{(\pi + \alpha\delta)} \tag{31b}$$

and

$$K_{t+1}^{**} = \frac{(1-\pi)\,\alpha\delta K_t^{\alpha}}{(\pi + \alpha\delta)} \tag{31c}$$

The Ricardian equivalence still holds along an equilibrium path with ultrarational agents, since the young anticipate that the optimal redistributive policy of the authority will be such that any change in the interest-bearing debt of the social security system will lead to an offsetting change in their future retirement benefits, leaving unchanged their total wealth. The consumption level of a young generation of ultrarational agents is lower than the level at which the young would consume if they believed in any feasible retirement scheme promised by the authorities ($C_{yt}^{**} < C_{yt}^{*}$). The obvious rationale for this result is that the young save more to support their old-age consumption when they anticipate that the transfers in their favour as old will be lower.

Consistently, the capital formation tends to be more intense with smart individuals than with naïve households ($K_{t+1}^{**} > K_{t+1}^{*}$); and the steady-state level to which the capital stock converges under ultrarational agents, $\bar{K}^{**}$, is such that $\bar{K}^{**} > \bar{K}^{*}$, where

$$\bar{K}^{**} = \left[\frac{(1-\pi)\,\alpha\delta}{\pi + \alpha\delta}\right]^{\frac{1}{(1-\alpha)}} \tag{32}$$

These results show that a public social security system, which is controlled by political authorities caring only about their popularity among the living generations, has long-term effects on capital accumulation and

on the standard of living of future generations which are mitigated if the private agents are smart. Conversely, the politicians managing social security would prefer to deal with naïve individuals, since – with the popularity of those controlling the system in t increasing with $w(C_{ot}, C_{yt})$ – we have $w(C_{ot}^*, C_{yt}^*) > w(C_{ot}^{**}, C_{yt}^{**})$. Furthermore, in case the political authority does not incur in any loss of popularity when the households realize that it is trying to "cheat" them, it is always convenient for the politicians to make the non-credible promises in (27) rather than announcing the only credible plan $\{V_{os,s}^*\}_{s=t}^{\infty}$, $\{V_{ys,s}^*\}_{s=t}^{\infty}$ and $\{B_{s+1,s}^*\}_{s=t}^{\infty}$. This is because, in the presence of some doubt about the ultrarationality of the households, it is solely in the "worst" case – the case in which the households are smart – that the non-credible announcement in (27) produces just the same level of $w(.)$ as the announcement of the credible plan.

In the present model there is a one-to-one substitutability in the portfolios of ultrarational agents only between non-pension and "true" pension wealth. The smart young living in t consider true pension wealth only that part of the present value of the promised social security benefits which is consistent with $V_{ot+1,t+1}^*$, that is with the transfers which it is rational to expect from the social security system during their retirement years. Any deviation of the promised pension benefits from $V_{ot+1,t+1}^*$ is not considered credible and does not induce any change in the demand for real and financial assets on the part of ultrarational agents. In other words, the degree of substitutability between promised pension wealth and non-pension wealth tends to zero as the promises concerning future social security benefits surpass the threshold beyond which it is reasonable to expect that they will not be honored: beyond this threshold, any further increase in the promised future benefits does not reduce the non-pension wealth.

4. *Final Remarks*

Relaxing the simplifying assumption of a stationary population would not lead to results that are qualitatively different from those exposed in this paper. It is probably more interesting to consider the case in which the political authorities reflect the preferences of individuals who are motivated by (at least) some concern for their descendants. Under these circumstances, we should expect announced policies treating the future generations more fairly.

The model is made more realistic by considering a population that consists of both naïve and ultrarational agents. The proportion of the house-

holds belonging to each group determines in each period the reaction of the private sector to the promises of the authority. These proportions can evolve over time according to what the former cohorts have experienced with respect to their promised pension benefits. Moreover, the distinction between those private agents who are able to skip the consequences of non-credible promises through their saving behavior and those who are not can be based on the difference in the endowments of private wealth and human capital that the individuals have at the beginning of their working life. Indeed, fixed costs associated with the acquisition of information and with the access to the capital market can prevent the poor from resorting to private schemes and getting alternative assets, even if they perceive that the public pension system is not totally reliable.

Even if the temporal horizon of a politician is too short to invest in building a reputation of reliability and credibility in the management of the social security system, we can think of households who "punish" the politicians making promises which are clearly non-credible. It is obvious that this possibility would amount to a restraint on the announcement of policies that are unlikely to be implemented. Symmetrically, it might be realistic to consider some sort of "punishment" for those politicians who do not keep the promises made to the retirees by a former generation of politicians. As a matter of fact, there may be naïve households that even if these promises were excessive and irrealistic based their saving behavior on them and are particularly damaged by their revision. This would make more difficult for the authority managing social security to adjust the parameters of the system according to its objectives.

APPENDIX

Under the assumption that the households are naïve and internalize any announced policy which is feasible, the problem of those managing the system in t is to find $\{V^*_{ot,t}\}_{s=t}^{\infty}$, $\{V^*_{yt,t}\}_{s=t}^{\infty}$ and $\{B^*_{s+1,t}\}_{s=t}^{\infty}$ which solve max $w(C_{ot}, C_{yt})$
subject to:

$$w(C_{ot}, C_{yt}) = \pi u(C_{ot}) + (1-\pi)\, u(C_{yt}) \tag{A1}$$

$$V_{os,t} + B_{s,t}\alpha K_s^{\alpha-1} = B_{s+1,t} - V_{ys,t} \qquad B_{t,t} = B_t \text{ given}, \qquad s = t, t+1, \ldots \tag{A2}$$

$$K_{s+1} = (1-\alpha)K_s^{\alpha} + V_{ys,t} - C_{ys} - B_{s+1,t} \qquad K_s \geq 0, \quad K_t \text{ given}, \qquad s = t, t+1, \ldots \tag{A3}$$

$$C_{os} = V_{os,t} + \alpha K_s^{\alpha-1}(B_{s,t} + K_s) \qquad C_{os} \geq 0 \qquad s = t, t+1, \ldots, \tag{A4}$$

$$C_{ys} = \frac{(1-\alpha)\,K_s^{\alpha} + V_{ys,t}}{(1+\delta)} + \frac{V_{os+1,t}\,K_{s+1}^{1-\alpha}}{(1+\delta)\,\alpha} \qquad C_{ys} \geq 0 \qquad s = t, t+1, \ldots \tag{A5}$$

$$\lim_{S \to \infty} \frac{B_{S,t}}{\prod_{j=t}^{S} \alpha K_j^{\alpha-1}} = 0 \tag{A6}$$

Notice that the problem of announcing an optimal plan in t under the assumption that the households are naïve is equivalent to the problem of finding an optimal policy under ultrarational agents, in case the authority which has control on the system in t can precommit those who will manage the system in the future to follow any guideline that it dictates in the present.

Rearranging (A3)-(A5), we obtain the condition that the households satisfy along an equilibrium path for optimal intertemporal allocation of consumption:

$$C_{yt} = \frac{C_{ot+1}}{\delta\alpha K_{t+1}^{\alpha-1}} \tag{A7}$$

Using (A2)-(A4), this condition can be rewritten as

$$C_{yt} = \frac{V_{ot+1,t}}{\delta\alpha\,(K_t^{\alpha} - C_{yt} - C_{ot})^{\alpha-1}} \div \frac{B_{t+1,t} + (K_t^{\alpha} - C_{yt} - C_{ot})}{\delta} \tag{A8}$$

The condition (A8) allows us to define the implicit functions

$$C_{yt} = f(C_{ot}, V_{ot+1,t}, B_{t+1,t}) \text{ and } C_{ot} = g(C_{yt}, V_{ot+1,t}, B_{t+1,t}).$$

Since $\frac{\partial f(.)}{\partial V_{ot+1,t}} > 0$, $\frac{\partial f(.)}{\partial B_{t+1,t}} > 0$, $\frac{\partial g(.)}{\partial V_{ot+1,t}} > 0$ and $\frac{\partial g(.)}{\partial B_{t+1,t}} > 0$, we have $\frac{\partial W(C_{ot}, C_{yt})}{\partial V_{ot+1,t}} > 0$ and $\frac{\partial W(C_{ot}, C_{yt})}{\partial B_{t+1,t}} > 0$. Considering (A2), (A3) and (A5), this implies that for those who manage social security in t it is optimal to set

$$V_{ot+1,t}^{*} = K_{t+1}^{\alpha} - A_{t+1}\,\alpha K_{t+1}^{\alpha-1} \tag{A9}$$

$$V_{yt+1,t}^{*} = -W_{t+1} = -(1-\alpha)\,K_{t+1}^{\alpha} \tag{A10}$$

and

$$V_{ot+2,t}^{*} = 0 \tag{A11}$$

Together with (A2)-(A6), (A9)-(A11) entail

$$V_{ys,t}^{*} = V_{os+1,t}^{*} = B_{s,t}^{*} = 0 \text{ for all } s > t+1 \tag{A12}$$

Substituting (A9) for $V_{ot+1,t}$ in (A8), we get

$$C_{yt} = \frac{K_t^{\alpha} - C_{yt} - C_{ot}}{\alpha\delta} \tag{A13}$$

Using (A1), (A4) and (A13), the problem can be reformulated as

$$\underset{V_{ot,t}}{\text{Max}}\ \pi \ln [V_{ot,t} + \alpha K_t^{\alpha-1}(K_t + B_t)] + (1-\pi)\ln\left[\frac{K_t^{\alpha} - V_{ot,t} - \alpha K_t^{\alpha-1}(K_t + B_t)}{1+\alpha\delta}\right]$$

The first order condition for a maximum is

$$\frac{\pi}{V_{ot,t} + \alpha K_t^{\alpha-1}(K_t + B_t)} - \frac{(1-\pi)}{K_t^{\alpha} - V_{ot,t} - \alpha K_t^{\alpha-1}(K_t + B_t)} = 0 \tag{A14}$$

from which we obtain

$$V_{ot,t}^{*} = \pi K_t^{\alpha} - A_t\, \alpha K_t^{\alpha-1} \tag{A15}$$

that gives

$$C_{ot}^{*} = \pi K_t^{\alpha} \tag{A16}$$

The equation (A15) can be used together with (A2) to obtain

$$V_{yt,t}^{*} - B_{t+1,t}^{*} = -(\pi - \alpha)\, K_t^{\alpha} \tag{A17}$$

Moreover, we get from (A13) and (A16) that

$$C_{yt}^{*} = \frac{(1-\pi)\, K_t^{\alpha}}{(1+\alpha\delta)} \tag{A18}$$

Under the assumption that the households are ultrarational and that the present planners are aware of their inability to precommit the future policy makers, the optimal strategy of the authority in t can be found by solving recursively the sequence of problems

$$\max w\,(C_{os}, C_{ys}),\ s = t, t+1, \ldots$$

subject to:

$$w\,(C_{os}, C_{ys}) = \pi u\,(C_{os}) + (1-\pi)\, u\,(C_{ys}) \tag{A19}$$

$$V_{os,s} + B_{s,s}\,\alpha K_s^{\alpha-1} = B_{s+1,s} - V_{ys,s} \qquad B_{t,t} = B_t \text{ given}, \tag{A20}$$

$$K_{s+1} = (1-\alpha)\, K_s^{\alpha} + V_{ys,s} - C_{ys} - B_{s+1,s} \qquad K_s \geq 0, \quad K_t \text{ given}, \tag{A21}$$

$$C_{os} = V_{os,s} + \alpha K_s^{\alpha-1}(B_{s,s} + K_s) \qquad C_{os} \geq 0 \tag{A22}$$

$$C_{ys} = \frac{(1-\alpha)\, K_s^{\alpha} + V_{ys,s}}{(1+\delta)} + \frac{V_{os+1,s} K_{s+1}^{1-\alpha}}{(1+\delta)\,\alpha} \qquad C_{ys} \geq 0 \tag{A23}$$

$$\lim_{S \to \infty} \frac{B_{S,S}}{\prod_{j=1}^{S} \alpha K_j^{\alpha-1}} = 0 \tag{A24}$$

We proceed by supposing that there is a period S, which is the final period of this economy. Considering that in S it is optimal for the young to set $A_S^{*}{}_{+1} = 0$, the condition that the authority must satisfy for a maximum is given by (A14) with $t = S$. From this condition, we obtain

$$V^*_{oS,S} = \pi K^\alpha_S - A_S \alpha K^{\alpha-1}_S \tag{A25}$$

that gives

$$C^*_{oS} = \pi K^\alpha_S \tag{A26}$$

Using (A20) and (A25), it is easy to check that

$$V^*_{yS,S} = -(\pi - \alpha) K^\alpha_S \tag{A27}$$

Taking into account (A20)-(A22), we can use (A26) to rewrite the condition in (A7) for $t = S-1$ as

$$C_{yS-1} = \frac{\pi (K^\alpha_{S-1} - C_{yS-1} - C_{oS-1})}{\alpha\delta} \tag{A28}$$

Using (A19), (A22) and (A28), the problem of the authority managing the system in $S-1$ can be reformulated as

$$\underset{V_{oS-1,S-1}}{\text{Max}} \quad \pi \ln [V_{oS-1,S-1} + \alpha K^{\alpha-1}_{S-1} A_{S-1}] +$$

$$+ (1-\pi) \ln \left[\frac{\pi (K^\alpha_{S-1} - V_{oS-1,S-1})}{\pi + \alpha\delta} - \frac{\pi\alpha K^{\alpha-1}_{S-1} A_{S-1}}{\pi + \alpha\delta} \right].$$

Solving this problem, we obtain

$$V^*_{oS-1,S-1} = \pi K^\alpha_{S-1} - A_{S-1} \alpha K^{\alpha-1}_{S-1} \tag{A29}$$

that gives

$$C^{**}_{oS-1} = \pi K^\alpha_{S-1} \tag{A30}$$

The equation (A29) can be used together with (A20) to obtain

$$V^*_{yS-1,S-1} - B^*_{S,S-1} = -(\pi - \alpha) K^\alpha_{S-1} \tag{A31}$$

Moreover, we get from (A28) and (A30) that

$$C^{**}_{yS-1} = \frac{(1-\pi)\pi K^\alpha_{S-1}}{(\pi + \alpha\delta)} \tag{A32}$$

Iterating this procedure i times, $S-i = t$, we find that

$$V^*_{os,s} = \pi K^\alpha_s - A_s \alpha K^{\alpha-1}_s \qquad s = t, t+1, \ldots \tag{A33}$$

and

$$V^*_{ys,s} - B^*_{s+1,s} = -(\pi - \alpha) K^\alpha_s \qquad s = t, t+1, \ldots \ldots \tag{A34}$$

are the policies which are optimal from the viewpoint of period s.

Notice that $\{V^*_{ys,s}\}^\infty_{s=t}$ and $\{B^*_{s+1,s}\}^\infty_{s=t}$ given by (A34) must be such that (A24) is satisfied along the associated perfect-foresight equilibrium path, which is characterized by (31) in the text.

Finally, one should consider that it is always convenient for the authority managing the system in t to announce the non-credible plan in (27), since *i*) for the current period this plan suggests the same redistributive policy suggested by (A33)-(A34), i.e. the optimal policy for period $s = t$ under ultrarational agents; *ii*) if the households are naïve, the announcement of (27) with its (non-credible) promises concerning the future will induce higher levels of consumption than the announcement of the credible plan, thus guaranteeing a higher return to the authority; *iii*) if the households are ultrarational, the promises that are implicit in (27) will not be taken seriously and the announcement of (27) will induce the same levels of consumption as the announcement of the credible plan (A33)-(A34), with no consequence on the popularity of the political authority.

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CREDIBILITÀ E POPULISMO NELLA GESTIONE DI UN SISTEMA PENSIONISTICO PUBBLICO

Le politiche pensionistiche pubbliche considerate devono essere fattibili, cioè coerenti con la solvibilità nel lungo periodo dell'ente che gestisce il sistema pensionistico, a cui sono consentiti squilibri temporanei tra entrate provenienti da contributi previdenziali ed erogazioni di benefici. I politici che sovraintendono alla gestione dell'ente sono incentivati a fare promesse non credibili, cioè promesse che per essere mantenute comporteranno un peso insopportabile per le generazioni future, se la loro popolarità cresce con i consumi delle generazioni viventi e se c'è la possibilità che gli individui credano che ogni politica pensionistica annunciata verrà effettivamente realizzata purché fattibile.

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THE PROBABILITY OF INSURANCE INSOLVENCY WITH LIMITED INSURANCE RESERVE

by
CHANDRASEKHAR MISHRA * and JORGE URRUTIA *

1. *Introduction*

Insolvency has become a serious problem in the insurance industry in recent years. More than 130 insurers failed in the last decade and a tendency to an increase in the number of failures has been observed. In effect, the year 1989 alone saw a record of 44 failures across the country, up from 31 in 1988. Some experts even suggest that the insurance industry is on the verge of a major financial crisis similar to the one experienced by the thrift institutions.

Insurers have aggressively pursued high risk underwriting and risky investment in junk bonds and real estate. Property-liability insurance companies who used to concentrate on predictable hazards, such as fire and theft, now underwrite far riskier lines, such as product liability and financial guarantees. Life insurers are acting more like commercial banks. Harrington and Nelson (1986) indicate that possible causes of the increase in insurers' insolvency are the managers' profit maximization behavior, difficulties in estimating incurred losses for riskier lines, and operation of the companies at higher levels of risk.

Academicians have long been concerned with the evaluation of the financial strength and the predictability of failure of insurance companies. Theoretical and empirical models have been derived and used to predict insurers' insolvency (Hershbarger and Miller, 1986; Ambrose and Seward, 1988). Multidiscriminant analysis has been applied by Hershbarger (1979). Portfolio analysis and the Capital Asset Pricing Model have been used to

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detect financially distressed insurers by Kahane (1978), Venezian (1983), Gustavson and Lee (1986), and Kahane, Tapiero and Jacque (1986). Logit models have been employed by Ohlson (1990) and Bar Niv and Hershbarger (1990). The overall prediction of these models have been weak and the general conclusion is that additional research is needed.

The purpose of this paper is to develop a theoretical model for insurance insolvency. The model determines the optimal level of an insurance reserve fund to be maintained by the insurer in order to avoid bankruptcy. The model also determines the perceived probability of insolvency of the insurer. It is found that the probability of insolvency is non-zero and it is minimized when the insurance reserve fund is set at the limiting level below which there is no excess funds to be returned to the policyholders. The remainder of the paper is organized as follows: Section 2 presents the derivation of the model for insurance company's insolvency; Section 3 derives a formula for the perceived probability of insurer's insolvency; Section 4 concludes the paper.

2. *A Model of Insurance Insolvency*

The model developed in this section is similar to the one in Tapiero and Jacque (1987), except that if there are excess funds, policyholders prefer the excess funds be returned to them in the form of lower premiums.

Let define TP as the total premiums collected by the insurance firm per period and x_t the aggregate claims at time t payable by the insurer. If the aggregate claims follow a lognormal distribution, then [1]

$$dx/x = \lambda dt + \sigma dz \tag{1}$$

where λ = mean rate of aggregate claims, σ^2 = variance rate of aggregate claims, and dz = standard Wiener increment.

Let us assume the insurer maintains an insurance fund, with a level y_s at time s, and that the funds are invested in risk-free assets to earn a rate of return r. The insurance system may become insolvent, but the probability of such event is not known in advance.

The level of the insurance fund changes for any small time interval dt,

[1] The lognormal distribution is widely used to represent claim distributions, because of its realistic shape; very small claims have small probability, medium sized claims are most frequent, large claims become less and less probable although even very large ones still have a positive probability of occurring.

as premiums are collected and claims are paid continuously [2]. The level of the insurance fund at time $s + dt$ is:

$$y_{s+dt} = y_s + TP\,dt - dx \tag{2}$$

By substituting (1) into (2), we have the following fund constraint [3];

$$dy = (TP - \lambda x)\,dt - \sigma x dz. \tag{3}$$

The insurance firm becomes insolvent when the level of the fund is inadequate to meet the claim obligations. If τ is the first instant of time the insurer is insolvent, then:

$$\tau = \inf\{t > s; y_t \leq 0\}\,^4 \tag{4}$$

where infinitum is over time t. Equation (4) corresponds to an insolvency constraint. When the insurer is insolvent, the outstanding claims at time τ are x_τ, and the insurance system is reorganized.

The optimization problem is to minimize the expected present values of the premium payments and the cost of bailout in the event the insurer is insolvent. If the insurance firm bankrupts, these costs must be borne by the policyholders. Let $J(y_s; \rho)$ be the discounted expected cost of the reserve fund, defined as the sum of the discounted cost of premiums and the discounted expected cost of bankruptcy. The minimization is subject to the fund constraint given by equation (3) and the insolvency constraint given by equation (4). Let ρ be the limiting level of the fund beyond which there are excess funds. If there are excess funds, policyholders prefer the excess funds be returned to them in the form of lower premium payments. Let K be the discounted expected claim obligations at the time of the

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[2] For simplicity, administrative costs are ignored as these costs are certain; the incorporation of these costs in the model will not change the conclusions.

[3] Since both the mean term $(TP - \lambda x)$ and the variance term $(\sigma^2 x^2)$ are necessarily continuous and bounded, the Ito stochastic differential equation (3) is well defined.

[4] Alternatively, let define an open set Ω as follows:

if $y \in \Omega$, process is continued, and
if $y \in \bar{\Omega}$, process is stopped.

The boundary to the open set, within which the process is defined is $\Gamma = \Omega \cap \bar{\Omega}$. In our example, $\Omega = \{y: y \geq 0\}$ and $\bar{\Omega} = \{y: y \leq 0\}$; thus, $\Gamma = \{y: y = 0\}$. Define τ as the first instant of time the process reaches the boundary Γ (from the initial time s). Thus, $\tau = \inf\{t \geq s; y_t \in \Gamma\}$.

insurer's insolvency [5]. K is a function of the premium rate received, the aggregate claims paid out, and the dividends paid out by the insurance firm, when y is in excess of ρ; thus, $K = K(\rho)$. Under these conditions, the insurer's optimization problem is:

$$\begin{aligned} &\underset{y_s}{\text{Min}}\, J(y_s; \rho) \\ \text{s.t. } &dy = (TP - \lambda x)\, dt - \sigma x dz \qquad (5) \\ &\tau = \inf\{t > s, y_t \leq 0\} \\ &y_s \leq \rho \,^{6} \\ &y_\tau = K \end{aligned}$$

where y_s and y_τ are boundary conditions.

Consider a small time interval $(s, s + \Delta t)$. Using Bellman's fundamental principle, the optimal expected present cost is the sum of the current cash flow and the discounted expected optimal cost at time $s + \Delta t$. Let $J^s = \underset{y_s}{\text{Min}}\, J(y_s)$. Then, from Bellman's principle, we have that:

$$J^s = E_s\left\{ \int_s^{s+\Delta t} e^{-r(t-s)}\, TP dt + e^{-r\Delta t} J^{s+\Delta t} \right\} \qquad (6)$$

where E is the expectation operator.

For small Δt, i.e., $\Delta t \to dt$, we have:

$$\lim_{\Delta t \to dt} E_s \int_s^{s+\Delta t} e^{-r(t-s)}\, TP dt = e^{-rdt}\, TP dt \qquad (7)$$

Since $e^{-rdt} \approx (1 - rdt)$, and using (4), we get:

$$J^s = (1 - rdt)\{TP\, dt + E_s J^{s+dt}\} \qquad (8)$$

By using a Taylor series expansion for $J(y)$, ignoring the higher-order terms of dt, using equation (3) and the properties of Ito calculus, and rearranging, we get:

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[5] See MERVILLE and PIEPTEA (1989) for an estimation of K when the firm's liquidation policy is exogenous.

[6] $y \leq \rho$; thus, ρ is the reflection boundary.

$$E_s J^{s+dt} = J^s + \frac{\partial J}{\partial y}(TP - \lambda x)\,dt + \frac{\partial J}{\partial t}\,dt + 0.5\,\frac{\partial^2 J}{\partial y^2}\,\sigma^2 x^2 dt \qquad (9)$$

By combining equations (6) and (9), and rearranging, we get:

$$J^s = (1 - rdt)\left\{ TPdt + J^s + \frac{\partial J}{\partial y}(TP - \lambda x)\,dt + \frac{\partial J}{\partial t}\,dt + 0.5\,\frac{\partial^2 J}{\partial y^2}\,\sigma^2 x^2 dt \right\} \qquad (10)$$

By ignoring higher order terms of dt, considering only stationary cost policies of $J(.)$, so that $\partial J/\partial t = 0$, and by writing $\partial J^s/\partial y_s$ as J_y, we obtain:

$$J = TP\,dt + J + J_y\,(TP - \lambda x)\,dt + 0.5\,\sigma^2 x^2 dt\,J_{yy} - rJ\,dt,$$

or

$$(TP - \lambda x)\,J_y + 0.5\,\sigma^2 x^2 J_{yy} - rJ + TP = 0 \qquad (11)$$

Equation (11) is a second order ordinary differential equation, which is subject to the following boundary conditions: $y_\tau = 0$, and $J(y_\tau) = K$. That is, when the insurer is insolvent, the insurance fund is wiped out and the discounted expected cost of the insurance equals the discounted expected value of the outstanding claim-demands at the time of insolvency.

Notice that ρ is the limiting level or the reflection boundary of the fund, beyond which the excess funds are returned to the policyholders in the form of lower premiums. Let $J(\rho)$ be the expected present cost when the limiting level of the fund is ρ. If the reflection is costless, then $\partial J/\partial y|_{y=\rho} = 0$, and the boundary conditions are [7]:

$$J(y = 0) = K \qquad (12)$$

$$J_y(y = \rho) = 0$$

The above second-order ordinary differential equation with two boundary conditions can be easily solved [8]. The homogeneous part of the differential equation (11) is:

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[7] When the reflection is costly, as long as we are on the boundary, the cost increases at a rate equal to the reflection cost, say α, or $\partial J/\partial y|_{y=\rho} = \alpha$ (TAPIERO, 1988). By ignoring dividend distribution costs, the present model assumes that the reflection is costless.

[8] The existence and uniqueness of the solution to the optimization problem in (5) is shown

$$(TP - \lambda x) J_y + 0.5\,\sigma^2 x^2 J_{yy} - rJ = 0 \tag{13}$$

The characteristic equation is $(TP - \lambda x)\,\phi + 0.5\,\sigma^2 x^2 \phi^2 - r = 0$, and the roots are:

$$\phi_1 = -\frac{TP - \lambda x}{\sigma^2 x^2} + \sqrt{\left[\left(\frac{TP - \lambda x}{\sigma^2 x^2}\right)^2 + \frac{2r}{\sigma^2 x^2}\right]} \tag{14}$$

and

$$\phi_2 = \frac{TP - \lambda x}{\sigma^2 x^2} - \sqrt{\left[\left(\frac{TP - \lambda x}{\sigma^2 x^2}\right)^2 + \frac{2r}{\sigma^2 x^2}\right]}$$

Clearly, $\phi_1 > 0$ and $\phi_2 < 0$. Thus, the solution to the homogeneous part of the differential equation is:

$$J = c_1 e^{\phi_1 y} + c_2 e^{\phi_2 y} \tag{15}$$

where c_1, c_2 are constants. Next, consider the particular part $(-TP)$ of the differential equation [9]. Thus, $J = A_0$ is the proposed solution to the particular part, where A_0 is a constant. To get A_0, differentiate the solution of the particular part; $J_y = 0$ and $J_{yy} = 0$. Substituting these partial derivatives of $J(.)$ into the differential equation (11) gives $A_0 = TP/r$. Therefore, the complete solution to the differential equation is the sum of the homogeneous and the particular parts:

$$J = c_1 e^{\phi_1 y} + c_2 e^{\phi_2 y} + TP/r \tag{16}$$

Applying the boundary conditions (12) to equation (16) yields:

1. at $y = 0, J = K$, gives

$$K = c_1 + c_2 + TP/r \tag{17}$$

2. at $y = \rho, J_y = 0$, gives

$$0 = c_1 \phi_1 e^{\phi_1 \rho} + c_2 \phi_2 e^{\phi_2 \rho} \tag{18}$$

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through an equivalent problem given by the ordinary differential equation (11) subject to the boundary conditions (12).

[9] The solution to the particular part of the differential equation is obtained by the method of undetermined coefficients. This method is initiated by guessing the form of the particular part up to arbitrary multiplicative constants. These arbitrary constants are then evaluated by substituting the proposed solution into the given differential equation and equating the coefficients of like terms (INCE, 1956).

From equations (17) and (18), we have:

$$c_2 = \frac{(K - TP/r)\,\phi_1}{\phi_1 - \phi_2\, e^{(\phi_2 - \phi_1)\rho}}$$

and

$$c_1 = \frac{(K - TP/r)\,(-\phi_2\, e^{(\phi_2 - \phi_1)\rho})}{\phi_1 - \phi_2\, e^{(\phi_2 - \phi_1)\rho}} \tag{19}$$

By substituting the values of the constants in equation (15), the complete solution J to the insurer's optimization problem is given by:

$$J = \frac{TP}{r}\left[1 - \frac{\phi_1\, e^{\phi_2 y} - \phi_2\, e^{(\phi_2 - \phi_1)\rho + \phi_1 y}}{\phi_1 - \phi_2\, e^{(\phi_2 - \phi_1)\rho}}\right] + K\left[\frac{\phi_1\, e^{\phi_2 y} - \phi_2\, e^{(\phi_2 - \phi_1)\rho + \phi_1 y}}{\phi_1 - \phi_2\, e^{(\phi_2 - \phi_1)\rho}}\right] \tag{20}$$

Let define

$$p = \frac{\phi_1\, e^{\phi_2 y} - \phi_2\, e^{(\phi_2 - \phi_1)\rho + \phi_1 y}}{\phi_1 - \phi_2\, e^{(\phi_2 - \phi_1)\rho}} \tag{21}$$

Then,

$$J = \frac{TP}{r}(1 - p) + Kp \tag{22}$$

Equation (22) is the solution to the minimization problem, and it is essentially the same result reported by Tapiero and Jacque (1987). Thus, the expected present cost for the insurer is given by the sum of two components: the first component is the present cost of the premium rate collected if the insurer doesn't default, the probability of which is $(1 - p)$; and the second component is the present cost of bailout in the event the insurer defaults, the probability of which is p. Equation (22) indicates that the higher the interest rate the lower the present value of the reserve fund maintained by the insurer.

3. *Probability of Insolvency*

A non-zero probability of insolvency for the insurance firm concerns not only the shareholders, who demand higher returns on their investments, but also creates structural incentive problems that ought to be examined in order to prevent the insurer's insolvency. Moreover, a non-zero probability of insolvency affects the premium negotiation between policyholders and

the insurer; it may also require a vulnerable insurer to obtain reinsurance. The probability of insolvency can be determined from a reservoir model where the inflows are premiums, that are deterministic and steady, and the outflows are claims, which are stochastic and subject to fluctuations. The level of the reservoir at current time corresponds to the initial reserves plus the accumulated premiums minus the accumulated claims.

From the model of insurance fund, the perceived probability of insolvency, p, is given in equation (21), with ϕ_1 and ϕ_2 defined in equation (14). The probability of insolvency is a function of several variables: (1) claim parameters, i.e., λ and σ; (2) the current level of aggregate claims; (3) the premium rate collected; (4) the market interest rate; (5) the limiting level of the insurance fund; and (6) the current level of the insurance fund.

Let $\phi_3 = \phi_2 e^{(\phi_2 - \phi_1)\rho}$, then

$$p = \frac{\phi_1 e^{\phi_2 y} - \phi_3 e^{\phi_1 y}}{\phi_1 - \phi_3} \tag{23}$$

The probability of insolvency is minimized when the current level of the insurance fund is set to the limiting level, ρ. The minimum insolvency probability at ρ is given by:

$$p_{y=\rho} = \frac{e^{\phi_2 \rho}(\phi_1 - \phi_2)}{\phi_1 - \phi_2^{(\phi_2 - \phi_1)\rho}} > 0 \tag{24}$$

Equation (24) also indicates that there is always a non-zero probability that the insurer will become insolvent.

4. *Concluding Remarks*

This paper develops a model for insurance insolvency by determining the optimal level of the insurer's reserve fund to be maintained in order to avoid bankruptcy. The insurer's optimization problem is solved by minimizing the expected present cost of premium payments and cost of bankruptcy subject to reserve fund constraint, insolvency constraint, and boundary conditions.

The paper determines the perceived probability of insolvency of the insurance firm. It is found that the perceived probability of insolvency is always positive and is minimized when the level of the insurer's reserve fund is set at the limiting level beyond which excess funds are returned to the policyholders.

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LA PROBABILITÀ DI INSOLVENZA NELLE ASSICURAZIONI CON RISERVE LIMITATE

L'articolo sviluppa un modello teorico per determinare la probabilità di in-

solvenza nelle assicurazioni quando le riserve sono limitate. La probabilità di insolvenza è una funzione di parecchie variabili: parametri sulle denunce, livello corrente delle denunce, premi raccolti, saggio d'interesse di mercato, livello limite delle riserve e livello corrente delle riserve. Questa probabilità è diversa da zero ed è minimizzata quando i fondi assicurativi sono posti al livello limite oltre il quale non vi sono fondi in eccesso da restituire ai titolari delle polizze.

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A FAMILY OF DECOMPOSABLE POVERTY INDICES

by
PANOS TSAKLOGLOU *

1. *Introduction*

Following the seminal article of Sen (1976), many attempts have been made to construct "distribution-sensitive" poverty indices. Identifying as poor each member of a given population whose income is less than or equal to the exogenously determined poverty line, z, it can be claimed that today there is a general agreement that a poverty index should satisfy the following axioms:

Focus axiom. Changes in the incomes of the non-poor which do not affect the number of poor should leave the index unaffected [1].

Monotonicity axiom. Ceteris paribus, a reduction in the income of a poor person should increase the index.

Transfer axiom. Ceteris paribus, a regressive transfer between two poor persons should increase the index.

Implicit in poverty measurement are three further axioms derived from the measurement of inequality. The *symmetry axiom*, which requires the index to remain unaffected by a permutation of incomes; the *income-unit independence axiom*, which requires the index to remain unaffected if the incomes of all the population members and the poverty line change by the same proportion; and the *population-size independence axiom*, which

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[1] Since the poverty line is assumed to be exogenous, the focus axiom does not rule out the use of the entire income distribution in fixing the poverty line.

requires the index to remain unaffected if two or more identical populations are pooled.

Sen (1976) further proposes the following general form for a poverty index [2] ($j = 1 \ldots q$)

$$P(z; \mathbf{y}) = A(z; \mathbf{y}) \, \Sigma_j (z - y_j) \, w_j(z; \mathbf{y}) \tag{1}$$

where $\mathbf{y}$ is the ranked income vector of the poor ($0 < y_1 \leq y_2 \leq \ldots \leq y_q \leq z$), $A(z; \mathbf{y})$ is a normalization factor, $z - y_j$ is the "poverty-gap" of person j, w_j is a weight attached to this poverty-gap and q is the number of the poor. As Hagenaars (1987) points out, essentially Sen's argument is that a poverty index should include three elements: the relative number of the poor (indicating the incidence of poverty), the shortfall of the poor from the poverty line (indicating their "absolute deprivation") and a weighting system of these shortfalls (indicating their "relative deprivation"). Then, using the notion of "ranked relative deprivation" and a normalization procedure requiring that if all the poor have the same income the index should be equal to the product of the "head-count ratio" $H = q/n$ (where n is the population size) and the "income-gap ratio" $I = \Sigma_j [(z - y_j)/qz]$, Sen derives the following index

$$P_S = 2\Sigma_j (z - y_j)(q + 1 - j)/[(q + 1) nz] \tag{2}$$

Taking into account that the Gini index for the distribution of income among the poor, G_p, is given by

$$G_p = 1 + 1/q - 2\Sigma_j y_j (q + 1 - j)/q^2 \mu_p \tag{3}$$

where μ_p is the mean income of the poor, it is not difficult to show that

$$P_S = H[I + (1 - I) G_p \, q/(q + 1)] \tag{4}$$

which for large q approaches [3]

$$P_S = H[I + (1 - I) G_p] \tag{5}$$

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[2] As FOSTER (1984) points out, SEN (1977) required A to be a function of the population size n, q and z. However, since n is assumed to be fixed and q is a function of $\mathbf{y}$ and z, it can be written as $A(z; \mathbf{y})$.

[3] It should be noted that although in his 1976 article Sen claimed that P_s satisfies the three axioms mentioned above, in a later work (SEN, 1977) he points out that P_S can violate the transfer axiom. This can happen if after the transfer one of the poor persons crosses the poverty line. For this reason SEN (1981) introduces the "*weak transfer axiom*" which adds the qualification "if both of them remain poor after the transfer" to the transfer axiom.

Several authors have suggested modifications and generalizations of P_S in order to incorporate in it various features or characteristics they consider desirable. Anand (1977) introduced variations of P_S by modifying its normalization scheme, Thon (1979) and Kakwani (1980) developed poverty indices by changing the weights of P_S, while Hamada and Takayama (1977) and Takayama (1979) introduced the concept of "truncated" distribution and derived poverty indices based on the Atkinson index of inequality [4]. An interesting feature of the contribution of Kakwani (1980) is the introduction of the following axiom.

Transfer sensitivity axiom. Ceteris paribus, in a ranked income distribution the transfer of a fixed amount of income between two poor persons a fixed number of ranks apart must have a larger impact on the index the closer the poor persons are to the bottom of the distribution.

P_S does not satisfy this axiom, whereas its modification introduced by Kakwani does.

2. *A New Class of Additively Decomposable Poverty Indices*

For policy purposes, a particularly desirable property is additive decomposability. A poverty index is additively decomposable if it satisfies the following axiom:

Decomposability axiom. If the population is grouped into k mutually exclusive and exhaustive groups the index must be equal to the weighted sum of the k group indices [5].

The head-count ratio, H, is an additively decomposable poverty index with population share weights, but it does not satisfy the monotonicity and transfer axioms. P_S and the indices related to it are not additively decomposable since they contain the Gini index for the distribution of income among the poor. The most widely used decomposable poverty index is that suggested by Foster et al. (1984)

[4] See also BLACKORBY and DONALDSON (1980) and CLARK, HEMMING and ULPH (1981). Further, KUNDU and SMITH (1983) introduced indices which satisfy the *proportion of the poor axiom* which states that "an increase in the relative number of the poor should increase the index". However, as demonstrated by the authors, no index can satisfy the axioms of population-size independence, proportion of the poor and transfer simultaneously.

[5] FOSTER and SHORROCKS (1991) introduce the *subgroup consistency axiom* which states that "Ceteris paribus, the poverty index should increase when poverty increases within a population subgroup". HAGENAARS (1987) calls this axiom *decomposition axiom*. All the additively decomposable poverty indices satisfy the subgroup consistency axiom but not vice versa.

$$P_F = (1/n)\,\Sigma_j\,[(z-y_j)/z]^{\varepsilon} \tag{6}$$

where ε is a measure of poverty aversion ($\varepsilon \geq 0$). P_F satisfies the focus axiom for all the values of ε, the monotonicity axiom for $\varepsilon > 0$, the transfer axiom for $\varepsilon > 1$, and the transfer sensitivity axiom for $\varepsilon > 2$. Foster et al. (1984) focus mainly on $\varepsilon = 2$. Even though in the latter case P_F fails to satisfy the transfer sensitivity axiom, it can be expressed according to the general form (5) suggested by Sen (1976) by replacing the Gini coefficient by the squared coefficient of variation among the poor.

Foster et al. (1984) claim that one of their main motivations for the construction of P_F is that it is closely related to the notion of relative deprivation. Foster (1984, p. 239) notes that "when the poverty line is taken as the reference point of the poor, the poverty gap corresponds very closely to the 'magnitude of relative deprivation' ". Two points can be made regarding this statement. Firstly, if the poverty line represents the minimum socially acceptable standard of living, then the difference between the poverty line and the poor individual's income – the poverty gap – represents his/her absolute rather than relative deprivation. Secondly, if a poverty index can be written in the general form (1) suggested by Sen (P_F can be written in that form) the poverty gap is already part of the index. Therefore, the notion of relative deprivation should be captured by the weighting scheme employed in the index. If the poverty line is taken as the reference point of the poor, a logical consequence of the above statement would be to use as weight for a poor individual's poverty gap not the poverty gap itself, but the ratio of the poverty line over the income of that individual. This leads to the following index

$$P_{T1} = (1/nz)\,\Sigma_j\,(z-y_i)\,(z/y_i) \tag{7}$$

P_{T1} belongs to a class of poverty indices which can be written according to the general form (1) as [6]

$$P_T = (1/nz)\,\Sigma_j\,(z-y_j)\,(z/y_j)^{\varepsilon} \tag{8}$$

where, as in the Foster et al. (1984) index, ε is a poverty aversion parameter ($\varepsilon \geq 0$); the higher the value of ε the higher the weight attached to the poorest individual.

Since P_T does not contain any expression related to the incomes of the non-poor, it satisfies the focus axiom. Taking into account that

[6] Note that both P_T and P_F belong to a more general class of poverty indices given by $P^* = (1/n)\,\Sigma_j f(x_j)$, where $x_j = (y_j/z)$. In the case of P_T $f(x_j) = (1-x_j)\,x_j^{-\varepsilon}$ whilst in the case of P_F $f(x_j) = (1-x_j)^{\varepsilon}$.

$$\partial P_T/\partial y_j = -(1/n)\,(z^{\varepsilon-1}/y_j^{\varepsilon+1})\,[\varepsilon\,(z-y_j)+y_j] < 0 \qquad (9)$$

P_T satisfies the monotonicity axiom for all the values of ε. In other words, an increase in the income of a poor person results in a decline in the value of the index. The second derivative of P_T with respect to y_j is given by

$$\partial^2 P_T/\partial y_j^2 = (1/n)\,\varepsilon\,(z^{\varepsilon-1}/y_j^{\varepsilon+2})\,[\varepsilon\,(z-y_j)+z+y_j] > 0 \qquad (10)$$

The same result holds for each y_j for $\varepsilon > 0$. Therefore, P_T is strictly convex to the incomes of the poor and, hence, satisfies the transfer axiom; that is, it attaches greater weight to the incomes of the very poor persons than to the incomes of the relatively better-off poor persons. Following Kolm (1976), a poverty index satisfies the transfer sensitivity axiom if its third derivatives with respect to the vector of incomes have the opposite signs than the second derivatives. Since

$$\partial^3 P_T/\partial y_j^3 = -(1/n)\,\varepsilon\,(\varepsilon+1)\,(z^{\varepsilon-1}/y_j^{\varepsilon+3})\,[\varepsilon\,(z-y_j)+2z+y_j] > 0 \qquad (11)$$

P_T satisfies the transfer sensitivity axiom for $\varepsilon > 0$. As a result, if a transfer takes place from the jth poor with income y_j to a poor with income $y_j + \delta$, then for a given $\delta > 0$, the magnitude of increase in P_T decreases as j increases. As Lambert (1993) points out, this implies that the poverty aversion of a poor individual is rising as his/her income declines.

Further, expressed in continuous form P_T can be written as

$$P_T = \int_0^z [(z-y)/z]\,(z/y)^{\varepsilon} f(y)\,dy$$

Then, since

$$\partial P_T/\partial z = \varepsilon z^{\varepsilon-1} \int_0^z y^{-\varepsilon}\,[(z-y)/z]\,f(y)\,dy + z^{\varepsilon-2} \int_0^z y^{1-\varepsilon} f(y)\,dy > 0 \qquad (12)$$

P_T is a strictly increasing function of the poverty line, z [7]. Although it seems natural that as the poverty line rises the value of a poverty index should rise, some poverty indices do not satisfy this property [8]. Thirdly, if the

[7] If $\partial P_T/\partial z$ is calculated from (8) the effect of the change in z on the number of poor, q, is ignored.

[8] For example, CLARK, HEMMING and ULPH (1981) note that at least the indices suggested by SEN (1976), TAKAYAMA (1979) and the first index suggested by CLARK, HEMMING and ULPH (1981) might not increase when the value of the poverty line increases.

population is grouped into $k = 1 \ldots K$ mutually exclusive and exhaustive groups with populations $n_1 \ldots n_K$, P_T can be written as

$$P_T = \Sigma_k (n_k/n) (1/n_k z) \Sigma_{j \varepsilon S k} (z - y_j) (z/y_j)^\varepsilon = \Sigma_k (n_k/n) P_{Tk} \qquad (13)$$

where P_{Tk}, n_k and S_k are, respectively, the level of poverty, the population size and the set of poor in group k. Therefore, P_T is an additively decomposable poverty index with population share weights and the quantities $(n_k/n) P_{Tk}$ and $100 (n_k/n) (P_{Tk}/P_T)$ can be considered as the absolute and percentage contributions of group k to aggregate poverty, respectively. Further, when $\varepsilon = 1$, P_T becomes a "constant elasticity" poverty index

$$(\partial P_{T1}/\partial y_j) (y_j/P_{T1}) = -z/\Sigma_j (z - y_j) = -z/q (z - \mu_p) \qquad (14)$$

This implies that, when $\varepsilon = 1$, equal *proportional* changes in the incomes of any pair of poor persons have exactly the same effect on P_T.

It was noted earlier that ε can be viewed as a "poverty aversion parameter". The larger its value the relatively larger the weight attached to the income-gap of the poorest poor. As ε tends to infinity P_T approaches a Rawlsian type of poverty index, focusing exclusively on the position of the poorest individual. At the other extreme, when $\varepsilon = 0$, P_T is equal to HI and violates the axioms of transfer and transfer sensitivity.

3. *An Empirical Illustration*

In this section estimates of P_T for different values of ε (0.25, 0.5, 1, 1.5, 2) are presented, along with estimates of H, I and the three other decomposable poverty indices that can be found in the literature. These are the index of Foster et al. (1984) which was cited above, the index of Chakravarty (1983) which is based on the notion of "utility gaps" $(0 < \varepsilon < 1)$

$$P_C = (1/n) \Sigma_j [1 - (y_j/z)^\varepsilon] \qquad (15)$$

and the index first suggested by Watts (1968) and re-introduced by Blackburn (1989) which is based on the Theil index of inequality

$$P_B = (1/n) \Sigma_j \ln (z/y_j) \qquad (16)$$

In line with most empirical applications of these indices, the value of $\varepsilon = 2$ is used in the case of P_F and the value of $\varepsilon = 0.5$ in the case of P_C. The data used are the consumption expenditure microdata of the 1981/82 Greek

Household Expenditure Survey. The distribution used is the distribution of equivalent consumption expenditure per capita [9]. In order to give an example of poverty decomposition, the population is divided into nine groups according to the type of profession of the household head. Following OECD (1976), the poverty line is fixed at the level of two thirds of the median equivalent consumption expenditure.

Estimates of the poverty indices along with each group's population share and the proportion of each group's poor with expenditures less than half the poverty line are reported in Table 1. The figures in square brackets give the poverty ranking of each group from the poorest [1] to the least poor [9]. The figures in round brackets reported below the decomposable indices give the percentage contribution of each group according to the relevant index. Several comments can be made regarding these results. All the indices, apart from I, suggest a similar ranking of the high-poverty groups. "Farmer", "Retired" and "Other" are respectively the three groups with the heaviest severity of poverty. On the other hand, there is no general agreement regarding the ranking of the low-poverty groups ("Professional and Technical worker", "Executive and Manager" and "Clerical worker"). The ranking of I is almost completely different from the ranking of all the other indices.

Turning to the results of decomposition analysis it can be noted that, as could be anticipated, as the value of ε in P_T increases, the contribution of groups with high income-gap ratios to the aggregate poverty burden rises. For example, as ε rises from 0.25 to 2 the joint contribution of the three groups with the highest income-gap ratios ("Farmer", "Retired" and "Other") increases from 66.5% to 71.1%. At the other end, the contribution of the population group with the lowest income-gap ratio ("Clerical worker") to the aggregate poverty is halved – from 1.6% it declines to 0.8% – as the value of the poverty aversion parameter ε rises from 0.25 to 2. In order to understand roughly the forces that drive the changes in the contributions of the population groups to the aggregate poverty as the value of ε rises, we can look at the estimates of the fourth column of Table 1 where the proportion of each group's "very poor" members is reported (proportion of poor falling below 50% of the poverty line). No member of the group "Clerical worker" falls below this threshold and, as a result, as the value of the poverty aversion parameter rises, its contribution to the aggregate poverty declines sharply. Similarly, relatively few members of the

[9] Description of the data and detailed discussion of aspects of poverty in Greece can be found in TSAKLOGLOU (1990).

TABLE 1
MEASUREMENT AND DECOMPOSITION OF POVERTY IN GREECE (1981/82)

Profession of Household Head	n_j/n	H	I	Proportion of poor below 50% of the poverty line	P_T ($\varepsilon = 0.25$)	P_T ($\varepsilon = 0.5$)
Professional or Technical worker	0.070	0.060 [8] (1.4)	0.239 [4]	0.097	0.017 [8] (1.7)	0.019 [8] (1.7)
Executive or Manager	0.018	0.056 [9] (0.4)	0.237 [5=]	0.250	0.016 [9] (0.4)	0.019[9] (0.4)
Clerical worker	0.059	0.093 [7] (2.4)	0.192 [9]	0.000	0.019 [7] (1.6)	0.021 [7] (1.5)
Sales worker	0.083	0.146 [6] (5.3)	0.224 [8]	0.046	0.036 [6] (4.4)	0.041 [6] (4.2)
Service worker	0.053	0.184 [5] (4.3)	0.237 [5=]	0.082	0.049 [5] (3.8)	0.056 [5] (3.7)
Farmer	0.183	0.378 [1] (30.5)	0.289 [2]	0.161	0.125 [1] (33.3)	0.146 [1] (33.6)
Production or Transportation worker	0.296	0.187 [4] (24.4)	0.235 [7]	0.105	0.050 [4] (21.5)	0.057 [4] (21.3)
Retired	0.156	0.345 [2] (23.8)	0.281 [3]	0.141	0.112 [2] (25.4)	0.130 [2] (25.6)
Other	0.082	0.189 [3] (6.8)	0.297 [1]	0.178	0.066 [3] (7.8)	0.077 [3] (8.0)
GREECE	1.000	0.227	0.264	0.129	0.069	0.079

groups "Sales worker", "Service worker" and, to a lesser extent, "Production or transportation worker" appear to live in conditions of severe deprivation and, hence, their contributions to the aggregate poverty decline as the value of the poverty aversion parameter increases. Exactly the opposite is the situation of the groups "Farmer", "Retired" and "Other" [10].

[10] It can be noted that according to the estimates of Table 1 the same argument applies to the contribution of the group "Executives and managers", as well. No great weight should be attached to this result since, taking into account the very small population share of this group, it may be attributable to measurement errors.

P_T			P_C ($\varepsilon = 0.5$)	P_F ($\varepsilon = 2$)	P_B
($\varepsilon = 1$)	($\varepsilon = 1.5$)	($\varepsilon = 2$)			
0.027 [8] (1.7)	0.041 [7] (1.9)	0.067 [7] (2.0)	0.0081 [8] (1.7)	0.0055 [8] (1.7)	0.018 [8] (1.7)
0.028 [7] (0.5)	0.041 [8] (0.5)	0.063 [8] (0.5)	0.0077 [9] (0.4)	0.0062 [7] (0.5)	0.018 [9] (0.4)
0.024 [9] (1.3)	0.028 [9] (1.1)	0.033 [9] (0.8)	0.0096 [7] (1.7)	0.0044 [9] (1.1)	0.021 [7] (1.6)
0.051 [6] (3.9)	0.065 [6] (3.5)	0.085 [6] (3.1)	0.0180 [6] (4.4)	0.0104 [6] (3.7)	0.040 [6] (4.3)
0.072 [5] (3.5)	0.096 [5] (3.3)	0.130 [5] (3.0)	0.0243 [5] (3.8)	0.0151 [5] (3.5)	0.054 [5] (3.8)
0.203 [1] (34.0)	0.291 [1] (34.5)	0.435 [1] (34.7)	0.0618 [1] (33.4)	0.0437 [1] (34.6)	0.041 [1] (33.5)
0.076 [4] (20.8)	0.105 [4] (20.2)	0.150 [4] (19.4)	0.0247 [4] (21.6)	0.0159 [4] (20.4)	0.055 [4] (21.3)
0.182 [2] (26.1)	0.265 [2] (26.8)	0.410 [2] (27.9)	0.0551 [2] (25.4)	0.0388 [2] (26.2)	0.126 [2] (25.5)
0.108 [3] (8.2)	0.157 [3] (8.4)	0.237 [3] (8.5)	0.0323 [3] (7.8)	0.0236 [3] (8.4)	0.074 [3] (7.9)
0.109	0.154	0.229	0.0339	0.0231	0.077

Further, these results suggest that the ranking and contributions to aggregate poverty of the various population subgroups according to P_C ($\varepsilon = 0.5$) and P_B are very similar to those of P_T for low values of ε (0.25, 0.5), while the ranking and percentage contributions of P_F ($\varepsilon = 2$) are very close to the corresponding rankings and contributions of P_T for $\varepsilon = 1$. Finally, the results of Table 1 illustrate some disadvantages of the index most widely used for poverty decomposition analysis (H). For instance, although H indicates that poverty is more acute in the group "Retired" than in the group "Production and Transportation worker", it also suggests that the contribution of the latter group to aggregate poverty is greater than the former group's contribution. However, as a result of a lower mean and a more unequal distribution of consumption expenditure among the former group's poor, the distribution-sensitive poverty indices P_T, P_C, P_F and P_B

indicate that the contribution of this group to aggregate poverty is substantially higher than the latter group's contribution. Moreover, the higher the value of the poverty aversion parameter ε in P_T, the larger the difference in the two groups' contributions to aggregate poverty appears to be.

4. *Conclusions*

The principal motivation of this paper is to provide an index which satisfies some desirable properties and, at the same time, corresponds to an intuitive notion of "poverty" more closely than existing indices. According to this notion of poverty, the relative deprivation of a poor person is derived by a direct comparison of his/her income with the poverty line. The relevant index and the family to which it belongs are derived, their properties are discussed and an application yields estimates and presents a comparison of this index with other indices used in the literature. Undoubtedly, many other approximations of the notion of relative deprivation can be presented and some of them can lead to the construction of decomposable poverty indices. In addition, like most other indices, the family of indices suggested here suffers from the disadvantage that it depends crucially on a poverty line which is arbitrarily exogenously determined. It is likely that future research will lead to the construction of decomposable poverty indices based on the notion of relative deprivation and using endogenously determined poverty lines.

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UNA FAMIGLIA DI INDICI DI POVERTÀ SCOMPONIBILI

È qui presentata una famiglia di indici di povertà scomponibili, che soddisfano i tre assiomi base di Sen (fuoco, monotonia, trasferimento) e possiedono ulteriori proprietà auspicabili (sensibilità al trasferimento, scomponibilità additiva, funzioni della linea della povertà strettamente crescenti). Un membro di tale famiglia è un indice di povertà ad elasticità costante. È poi presentata un'applicazione impiegando dati greci.

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FORMALIZING THE INFORMAL SECTOR: A COMPARATIVE ANALYSIS OF ASIAN AND AFRICAN COUNTRIES

by
YONG HE *

Introduction

The informal sector plays an important role in developing countries. According to ILO (International Labor Office)'s estimation over 17 African countries, the informal sector's production represents on average 20% of GDP, and 59% of urban labor work within the informal sector (cf. World Bank, 1989, chapter 6).

Anderson (1982) distinguished three phases of industrial development: 1. a phase in which household manufacturing is predominant. 2. a phase in which small workshops and factories increase at a comparatively rapid rate. 3. a phase in which large-scale production becomes predominant. We define the formalization of the informal sector as the passage from the first to the second phase described by Anderson.

This process of transformation is sometimes presented as unavoidable. To criticize the judgement that the informal sector only signifies a prolonged survival in times of crisis and can never found a development dynamics, Judet (1989) wrote: "we have forgotten to inquire the way by which industries as a whole have been formed with the passing decades, going from simple to complex, from unstructured to structured". However it seems that this argument only suits well with the experiences of Asian countries. As Anderson (1982) has shown, the countries of South-East Asia had rapidly realized their transformation from the first to the second phase as he defined.

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On the contrary, this transformation appears to be very difficult in the case of African countries. According to ILO (1989), while the proportion of the informal sector in total urban employment of African countries was on average 60% in 1970s, it remained at the same level in 1988 (58%). The World Bank (1989) recognized that, apart from the informal sector and large firms, African countries lack small and middle-size firms. This problem of "missing middle" is considered as the major obstacle of African industralization [1]. Rather than accumulate their capital, the most serious problem for informal enterprises is how to survive. According to a survey of a large number of informal enterprises in four African cities (Bamko, Lomé, Nouakchott and Yaoundé), there were 13,3% of employers and 83,7% of employees whose weekly wages were lower than the guaranteed Minimum Wage of their countries (Maldonado, 1987, p. 133).

To explain the difficulty of such a transformation from informal to formal enterprises, the supply side conditions are often emphasized. To Aftab et Rahim (1989), "barriers" to the growth of informal enterprises lie in their artisan base which is initially determined by the educational background, knowledge, skills and the economic resources of the owner-manager. So often the governments are appealed to improve their supply conditions: financial and technical assistance, formation, technology, infrastructure, input supply, etc. (Cf. Page and Steel, 1986).

However, in the Southeast Asian countries where informal enterprises are dynamic, the governments have not deliberately helped them in this manner. While, as several studies have shown, the policies in favor of the informal sector practised in India have led to failure. These observations lead us to follow Schmitz (1982, p. 445)'s reasoning: "the issue is not whether small enterprises have growth and employment potential but under what conditions". In other words, to us the supply side conditions may not be essential for their transformation.

This paper will analyze three markets: the labor market, the product market and the financial market. It tends to show that: 1. the difference between the performances of informal enterprises in Asian and African countries lies in their different rural conditions; 2. this difference links less with the supply side conditions, but more with the demand side conditions of informal enterprises. From this point of view, state intervention should stress the improvement of economic environment and the demand side conditions of informal enterprises.

[1] In the case of Kenya, cf. LIVINGSTONE (1991).

I. *Analytical Framework*

In this section, we will analyze how the performance of informal enterprises is conditioned by the labor market, the product market and the financial market.

I.1. *Labor market.* – Here we borrow a diagram from Hemmer and Mannel (1989) with some simplifications, which helps us to understand the interactions among the informal sector, the formal sector and the agricultural sector in the labor market.

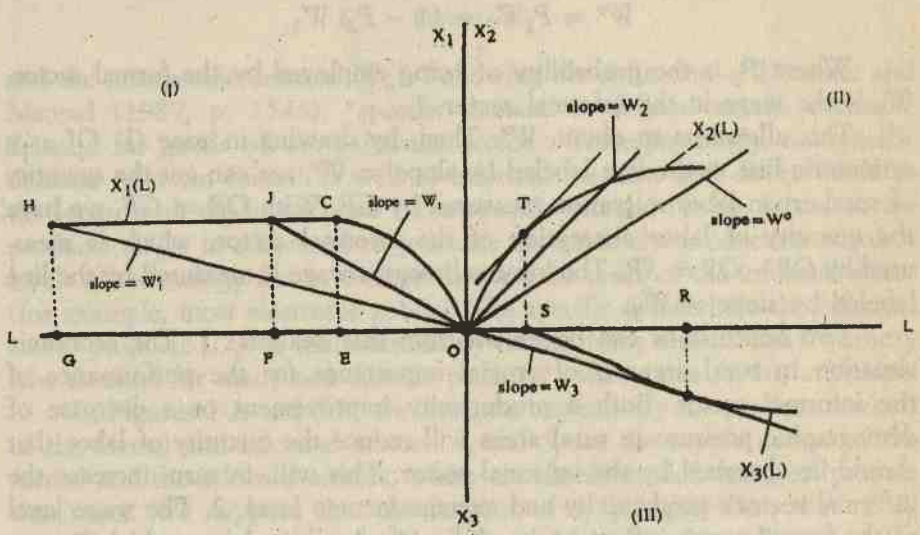


FIGURE 1.

In Figure 1, the horizontal axis represents the quantity of labor, the vertical axis the production levels of the agricultural sector (X_1), of the formal sector (X_2) and of the informal sector (X_3). We suppose that the total labor input is measured by OG. We also suppose that the production of each sector is only a function of labor input. So, for example, the production function of the agricultural sector is noted by $X_1(L)$. In zone (I), starting from point C, the slope of $X_1(L)$ equal to zero illustrates the existence of an excess rural labor input measured by EG. The average product (or the average income) of rural workers is measured by the slope of line OH.

In zone (II), $X_2(L)$ is the production function of the formal sector.

The slope of line W_2 represents the wage in the formal sector, which is often over-determined by public regulations related to the market-clearing wage. The tangent point between $X_2(L)$ and the line labeled by slope $= W_2$, T, determines the quantity of labor absorbed by the formal sector, which is measured by OS.

In zone (III), $X_3(L)$ is the production function of the informal sector. The labor absorption of the informal sector is determined in the following way. Based on Todaro (1969)'s model, the immigration decision of a rural laborer depends on his estimation concerning the wage he will receive after migration. This estimation (W^e) is calculated according to the formula

$$W^e = P_2 W_2 + (1 - P_2) W_3$$

Where P_2 is the probability of being employed by the formal sector. W_3 is the wage in the informal sector.

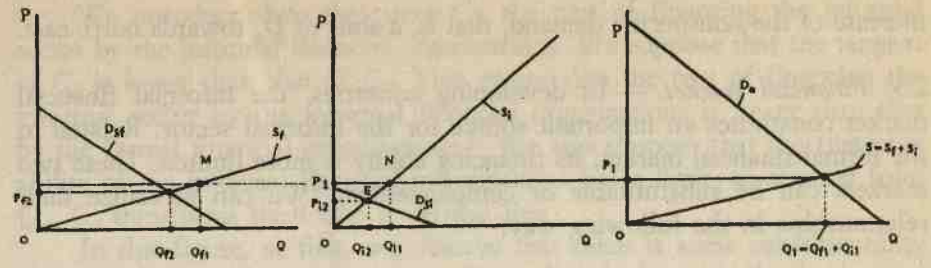
This allows us to obtain W^e. Then, by drawing in zone (I) OJ as a symmetric line to the line labeled by slope $= W^e$, we can get the quantity of rural-urban labor migration measured by GF. With $OR = GF$, we have the quantity of labor absorption of the informal sector, which is measured by $OR - OS = SR$. The informal sector's wage is measured by the line labeled by slope $= W_3$.

Two conclusions can be drawn from this analysis: 1. The economic situation in rural areas is of crucial importance for the performance of the informal sector. Both a productivity improvement or a decrease of demographic pressure in rural areas will reduce the quantity of labor that should be absorbed by the informal sector. This will, in turn, increase the informal sector's productivity and average income level. 2. The wage level of the formal sector influences its ability of absorbing labor, which in turn affects the pressure of employment induced by the informal sector. Furthermore, it modifies rural-urban migration through its impact on the income expectation of rural workers [2].

I.2. *Product market.* – We have shown how in the labor market the division of labor is realized among the agricultural sector, the informal sector and the formal sector. Now, we analyze how the product market shares are determined between the formal and informal sectors.

In Figure 2, we start by distinguishing the specific demand (D_s)

[2] The regulation of the wage level in the formal sector results in an increase in labor and a decrease in wages in the informal sector. These effects were formalized in RAUCH (1991)'s model.



(A) (B) (C)

FIGURE 2.

and the nonspecific demand (D_n). Following the definition of Hemmer and Mannel (1989, p. 1546), "specific demand is defined as the exclusive demand for goods of one particular sector... Correspondingly, nonspecific demand refers to formal as well as informal products and directs itself to the sector with the lower price". The specific demand for the formal sector products (D_{sf}) is addressed to products of which the production requires levels of technology and investment beyond the reach of the informal sector (for example, most electronic goods). The specific demand for the informal sector products (D_{si}) is characterized by its flexibility and its small quantity (the demand for many non-durable products corresponds to this criterion).

In Figures 2-A and 2-B, we illustrate respectively the product markets of the formal and informal sectors. S_f and S_i are the supply curves of the formal and the informal sectors, which are based on their marginal production costs. We suppose that the tangent of S_i is higher than that of S_f.

The intersections between S_i and D_{si}, and between S_f and D_{sf} determine the optimal specific productions of the formal sector (Q_{f2}) and of the informal sector (Q_{i2}), and their respective prices (P_{f2} and P_{i2}). We observe that if the informal sector's production cost is given, to increase its benefit, that is, to enlarge the triangle OEP_{i2}, the specific demand for its products should increase, that is, D_{si} should move towards north-east.

In Figure 2-C, S being the total marginal cost ($S = S_f + S_i$), its intersection with the nonspecific demand (D_n) gives the optimal nonspecific production (Q_1) and its optimal price (P_1). The formal sector and the informal sector respectively produce Q_{f1} and Q_{i1}, which can be observed in 2-A and 2-B, with $Q_1 = Q_{f1} + Q_{i1}$.

From this analysis of nonspecific product market, we conclude that the cost function of the informal sector being unchanged, the only way for it to increase its benefit (which is measured by ONP_1 in 2-B) is the

increase of the nonspecific demand, that is, a shift of D_n towards north-east.

I.3. *Financial market.* – In developing countries, the informal financial market constitutes an important source for the informal sector. Related to the formal financial market, its financing ability is more limited. These two markets can be substitutable or complementary. We can formalize their relationships in the following way:

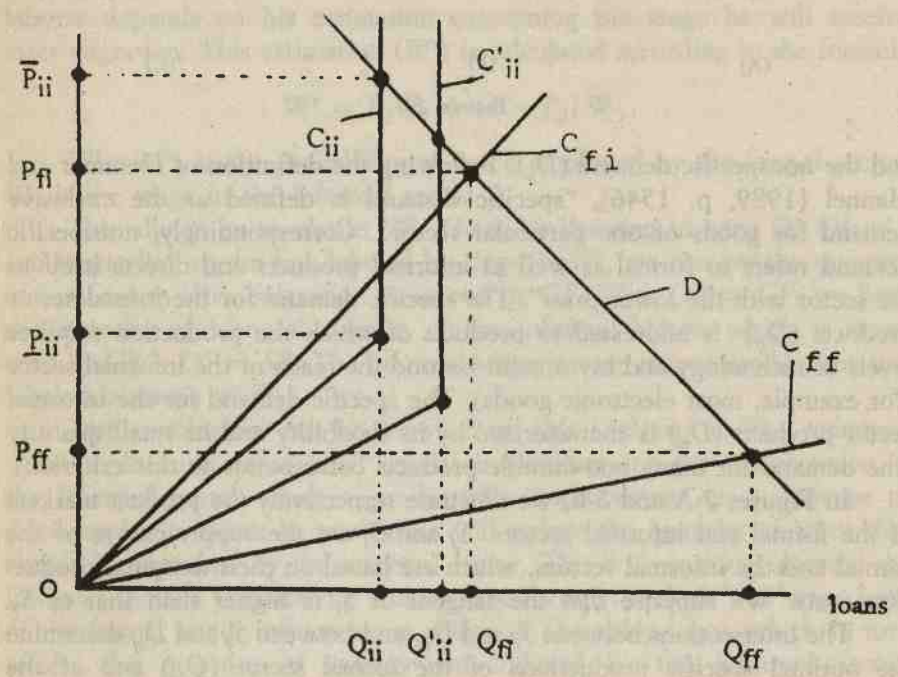


FIGURE 3.

In Figure 3, D is the demand curve for credits. For the formal financial organizations, their credits to the formal sector are less risky and then cheaper than those to the informal sector. This is reflected by the curve C_{ff} (the cost of financing the formal sector by the formal financial organizations), whose tangent is lower than that of the curve C_{fi} (the cost of financing the informal sector by the formal financial organizations). Consequently, the quantity of its loans to the informal sector (Q_{fi}) is much lower than that to the formal sector (Q_{ff}). Correspondingly, its interest rate for the informal sector (P_{fi}) is much higher than that for the formal sector (P_{ff}).

We introduce then the curve C_{ii}, the cost of financing the informal sector by the informal financial organizations. We suppose that the tangent of C_{ii} is lower than that of C_{fi}. This means that the cost of financing the informal sector by the informal financial organizations is lower than that by the formal financial organizations [3]. We also suppose that the financing ability of the informal financial organizations has a limit, so starting from Q_{ii}, C_{ii} transforms itself into a vertical line.

In this figure, at first, we observe that there is some substitutability between the two financial markets. Even though theoretically the interest rate of the informal financial market can vary between P_{ii} and $\bar{P}_{ii}$, it cannot be too speculative in that it cannot surpass P_{fi}, the interest rate of the formal financial market to the informal sector. Otherwise, the informal enterprises will turn to the formal financial market. Therefore there exists some degree of competition between the two financial markets, which prevents to some extent the interest rate to be too speculative in the informal financial market. Secondly, we can conclude that if the informal financial market improves its performance, that is, if the tangent of its cost curve reduces and (or) if it increases its financing ability, it can increase its quantity of loans with a lower interest rate. This is shown by the curve C'_{ii}, which leads to Q'_{ii}, with $Q'_{ii} > Q_{ii}$.

II. *Applications to Asian and African Countries*

II.1. *Rural Conditions.* – In our analysis of the labor market, we have shown how the performance of informal enterprises is conditioned by rural development, more exactly, by agricultural growth and population growth. From this point of view, the difference is fundamental between Asian and African countries.

At first, considering some Asian countries, we can observe a sustained agricultural growth during their industralization. On the contrary, in African countries, rural development remains a serious problem. In Africa, the population growth rate is over 3%, much higher than that of most developing countries (Cf. World Bank, 1989, Introduction). In Sub-Saharan Africa, agricultural growth for the period 1973-84 in real terms was only 0,7% per

[3] Because: 1. informal enterprises being small-sized, there can be no effect of the economy of scale for the formal financial organizations; 2. the informal financial organizations being near to their debtors, their costs of information are relatively low; 3. in general the informal financial organizations benefit from a trust link with their debtors, so the risk of their credits is lower. For more complete analysis, cf. GHATE (1992).

TABLE 1

GROWTH RATE OF AGRICULTURAL PRODUCTION (%)

Taiwan	1960-69 4,6	1970-79 4,5
Korea	1961-70 3,5	1971-85 3
	1971-81	1980-84
Indonesia	3,7	4,6
Malaysia	4,7	3,4
Philippines	4,9	2,1
Thailand	6,0	3,4

SOURCE: *Taiwan Statistic Data Book*, 1990; *Korea Statistic Yearbook*, 1971; *Korea Statistic Data Book*, 1990; BARGHOUTI, TIMMER and SIEGEL (1990, p. 15).

year, that is, – 1,9% per year and per inhabitant (Serageldin, 1989, p. 12).

Furthermore, Asian rural development is characterized by the expansion of non-farm activity, which may reduce immigration into towns. In Thailand, in 1960, the share of agriculture in total GDP was estimated at about 39,8%. This share was reduced to 17,5% in 1985. However, the reduction of rural population's share in total population was not at the same speed. It was reduced from 82,4% to 68,4% in the same period (Krongkaew, 1988, pp. 73-75). The explanation seems to be the rapid development of non-farm activity in the rural regions. In Thailand, by 1984, 34,7% of the total income of farm households is derived from non-farm activities. In Korea in 1979, non-farm employment accounted for nearly 65% of the total employment in rural areas. In Malaysia in 1978, rural non-farm activity provided about 32% of the primary employment in rural areas. In the Philippines, rural non-farm activity accounted for about 55% of the total income of rural households in 1971 (Cf. Mukhopadhyay and Lim, 1985, p. 7, p. 386).

This is not the case for African countries. For example, according to Livingstone (1991), in Kenya, the informal sector is heavily concentrated in towns. According to the World Bank (1987), in Kenya, for the period 1978-85, the elasticity of the informal sector employment with respect to GDP was 2,87, while it was only 0,49 for the private formal sector. According to Livingstone (1991), in Kenya, the informal sector employment has grown at 11,1% per year over the period 1985-88, while this growth

rate was only 2,7% in the private formal sector. In manufacture, these rates are respectively 15% and 2,3%.

All these observations support the conclusion: "the good health of agriculture is perhaps one of the most important factors of Asian industrial success in the past, while the difficulties in African agriculture constitute a further handicap for its industries". (Chaponnière and Judet, 1992).

II.2 *Market Size and Exportation.* – In our analysis of the product market, we have shown how the growth of informal enterprises is conditioned by the specific and nonspecific demands for the products of the informal sector. This demand condition is also very different between the Asian and African countries.

Clapham (1985, p. 21) affirmed that the size of consumer goods markets in developing countries is limited by low average incomes, inadequate transport possibilities and the specificity of the demand. Consequently there exist many small submarkets which are limited in volume and are frequently found at the local level.

This judgement seems more relevant for the informal sector in African countries. Based on some observations in Transkei in South Africa, Nattrass (1987) concluded that it is absurd to think that street traders could become the "giants of tomorrow's economy". The failure to succeed is the result of structural limitations, that is, the size and nature of the lower-income market. According to Livingstone (1991), in African countries, the dichotomy between the formal and informal sectors arises out of a corresponding income dichotomy between the mass of consumers making use of informal sector goods and services and a wealthy class largely patronizing the formal sector. However in many Asian countries there is more of a spectrum than a dichotomy [4].

What the African countries should learn from Asian countries is how their informal enterprises have utilized their comparative advantages to conquer the domestic markets and export markets. As Berry and Mazumdar (1991) affirmed, small-size firms play a significant role in exportation because their products are labor-intensive and they have the capacity to respond quickly to the changing conditions of markets. A representative survey undertaken in the early 1970s indicates that in Singapore, the share of output exported was 19,3% for the firms with fewer than 10 workers.

[4] The difference between rural population densities in African and Asian countries also plays a significant role. In the Island of Java, an informal entrepreneur can reach a market of 1,2 million consumers within a 20 Km radius, whereas in many African countries, he would have to travel more than 100 Km to meet the same population.

Chou (1988) has shown that the market structure in Taiwan is characterized by the fact that in the domestic market, there were relatively large-scale manufacturers with numerous small-scale distributors; in the export market, there were relatively small-scale firms with an enormous number of small-scale trading companies. According to Régnier and Kirchbach (1992), the share of exportation in the total production of small and middle enterprises accounts for 65% in Taiwan, 35% in Korea and 17% (direct exportation only) in Hongkong.

Following the Asian countries' example it seems indispensable for African countries to formalize their informal enterprises. Based on a survey undertaken in six African countries, Marsden (1990) has concluded that informal enterprises in Africa can be transformed into modern enterprises. But nearly all African enterprises he has taken as examples are exportation-oriented.

Another point to indicate is that the complementarity between small and large-size firms is one important reason for the small enterprises' success in Taiwan and in Korea (Cf. Ho, 1980, pp. 17-19). Informal enterprises appear to be efficient only in those industrial sectors where they do not meet the competition from modern large-size firms (Cf. Bruch and Hiemenz, 1984, pp. 44-52). Often small enterprises are observed to be exploited by the large ones in developing countries (Cf. Schmitz, 1982, pp. 433-7). However, as Amsden (1991) has argued, in Taiwan, large firms constitute an important support for small firms in the domain of credit and transfers of technology.

II.3. *Financial Conditions.* – In our analysis of the financial market, we have shown that the growth of informal enterprises is conditioned by the expansion of the informal financial market because the formal financial organizations, based on credit cost evaluation, always give a preference to formal enterprises.

In most Asian countries where small-size firms are very dynamic, large-size firms are always privileged in credit and in the access to raw materials (Cf. Berry et Mazumdar, 1991). In Taiwan, small-size firms have been discriminated against by the financial system. Three-quarters or more of what they lend must be secured by collateral. The reasons for this discrimination appear to be related to risk and transaction cost minimization (Cf. Amsden, 1991).

In the case of the informal sector, most of financial needs seem to

come from the informal financial markets [5]. Several scholars (McLeod, 1984; Yotopoulos and Floro, 1991), taking Indonesia and the Philippines as examples, have even concluded that in Asia, the informal financial system runs so well that the financial constraints of the informal sector do not represent a serious problem to its development any longer.

However, if we consider the African countries (such as Nigeria, Ghana, Tanzania, Uganda), we can observe the same tendency (Cf. Page, 1979, p. 21). The use of such systems as tontine or other forms of rotative associations in saving and in credit is very extensive [6]. In Kenya, the resources coming from personal savings and from families finance 92% of small-size enterprises' investments. In Zaire, this proportion attains 99% (World Bank, 1989, chapter 6).

So it seems that it is not the intensive use of informal financial markets, but the absolute size of the informal financial markets which makes the difference between Asian and African countries. The informal financial market size is tightly linked with the level of domestic savings. The latter is, in turn, conditioned by the general economic performance of the country. In Sub-Saharan Africa, the gross domestic saving represented 19,6% of GDP during 1975-80, it was 13,9% during 1981-85 and 11,8% during 1986-87 (World Bank, 1989, chapter 8), while this rate was on average 17,2% during 1961-70 and 23,7% during 1971-80 for the Philippines, Malaysia, Indonesia and Thailand. It was 17,2% during 1961-70 and 26% during 1971-80 for Taiwan, South Korea, Singapore and Hongkong (Kim, 1982, p. 15). If one takes into account the absolute levels of GDP, the difference of the credit ability of the informal financial markets between the Asian and African countries should be more significant.

III. *Implications to Governmental Policies*

The governments of developing countries are often solicited to help their informal enterprises. This is doubtlessly right. The question is how to help them? The experiences of Asian countries are instructive.

Ho (1980, p. 88) has argued that during their industrialization, neither the government in Korea nor the one in Taiwan actively promoted the development of small firms. Rather small firms were left largely to fend for

[5] For an estimation over the size of the informal financial markets in ten Asian countries, cf. GHATE (1992).

[6] For several studies about this subject, cf. HÉNAULT and M'RABET (1990, Part 2).

themselves. However, they helped determine the economic environment within which small firms operated.

On the contrary, in India, the economic policy of the government has deliberately favored small-size firms, not because they are more efficient, but because they are small. The results are naturally disappointing. The best example is that it used fiscal measures to encourage the development of handloom weavers and to brake the development of powerlooms. In spite of this, three quarters of the growth of cotton production came from powerlooms (Cf. Little, Mazumdar and Page, 1987; Kashyap, 1988).

The lack of human capital in informal enterprises is often used as an argument for governmental intervention. However, certain estimations showed that this is not a very serious problem for the growth of informal enterprises. According to Vijverberg (1991)'s calculation about the informal sector in Côte d'Ivoire, there is only weak evidence of returns to human capital. Even those of education are statistically insignificant. An empirical study undertaken by Nee and Young (1991) over some Chinese villages showed that the village education has no significant effect on the level of peasant entrepreneurship across villages.

In most of Sub-Saharan African countries, the governmental interventions seem to be undertaken in an inadequate way. For example, the exchange rate has been over-evaluated. This explains the loss of their export market shares (World Bank and UNDP, 1989). According to Balassa (1990), in these countries, a 1% change in the exchange rate is negatively associated with a 0,8-1% change in the ratio of exports to output.

Another example is price discrimination against agriculture. According to Cleaver (1985), in Sub-Saharan Africa, countries with a high degree of price discrimination against agriculture had an average agriculture growth rate of 0,8% in the 1970-81 period, while the corresponding growth rates were 1,8% to 2,9% in countries with medium and low price discrimination against agriculture.

Excessive bureaucratic interventions can also have a negative effect to the growth of informal enterprises. De Soto (1989) argued that the excessive regulation imposed by the enormous bureaucratic state was effectively excluding Peru's informal workers from full participation in the country's economic life. According to Nee and Young (1991), in Chinese villages, the most important negative determinant to the development of the peasant household firms is bureaucratic mobilization. Based on a survey in Tanzania, Levy (1991) concluded that small firms bear a punitive formal tax and licensing requirements, and that furthermore, these give opportunities to bureaucrats to get side payments.

With all these arguments, we agree with Little (1988)'s judgement: a policy of supply-side interventions in favor of small-scale enterprises will not promote an efficient labor-intensive development. The elements of a sound policy include the following: 1. Trade and protectionist policies should be changed to make exporting as profitable as domestic sales. 2. Interest rates, including those to small enterprises, should not be subsidized. 3. High wages in the formal sector encouraged directly or indirectly by governments should be abandoned or at least modified. 4. Any supply-side industrial promotion measures should disregard and, as far as possible, be neutral as to firm or establishment sizes.

This does not mean that the government has nothing to do in the informal entrepreneurship promotion. As Chickering and Salahdine (1991) have suggested, governments should eliminate institutions and policies that discriminate against the informal sector, such as difficult procedures to register or operate a business, to establish title to land, or to get a permit to build; burdensome regulations on economic activity; and policies that indirectly discriminate against the informal sector such as promotion of large enterprises, strict interest rate regulations, and trade restrictions.

Therefore the policy reform seems to be more important for most African countries. According to the World Bank and UNDP (1989, chapter 6), among the Sub-Saharan African countries, those that have actively taken part in reforms (over the exchange rate, the prices of agricultural products, and the public expenditure and public enterprises) have a much better economic performance than those that have not been active in these reforms. This is shown in Table 2.

TABLE 2

Indicator	Period	All countries	
		with a program and active in reforms	no program or weak effort in reforms
Agricultural Production	1980-84	1,1%	1,3%
	1985-87	2,6%	1,5%
Growth in volume of exportation	1980-84	– 1,3%	– 3,1%
	1985-87	4,2%	0,2%

Conclusion

We have used several simple analytical tools to show that (1) on the labor market, the formalization of the informal sector depends on the average income and the quantity of rural labor and on the wage level regulated in the formal sector; (2) on the product market, it depends on the market size; (3) on the financial market, it is linked with the financial ability of the informal financial sector, which is also determined by the size of the product market. We have shown that from these three points of view the conditions for the informal sector promotion are fundamentally different between Asian and African countries.

The implication of our analysis for the policy-making of governments in developing countries is that they should make efforts to develop their agriculture, to reform their wage policy vis-à-vis the formal sector, to reform the fiscal and exchange rate policies to promote exportation, and to reduce administrative burdens on informal entrepreneurs. In a word, improving the economic environment and the demand side conditions may be a more efficient way to promote the formalization of the informal sector.

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YOTOPOULOS Pan A. and FLORO Sagrario L., *The Informal Rural Credit Markets: An Application of the New Institutional Economics in the Philippines*, Boulder: Westview Press, 1991.

LA FORMALIZZAZIONE DEL SETTORE INFORMALE: UN'ANALISI COMPARATA TRA PAESI ASIATICI E AFRICANI

Analizzando il mercato del lavoro, quello dei prodotti e il mercato finanziario, si sa, in primo luogo, che la differenza tra le performance delle imprese informali nei Paesi asiatici ed africani risiede nelle diverse condizioni rurali; in secondo luogo, che tale differenza è connessa alle condizioni dell'offerta in misura minore che alle condizioni della domanda delle imprese informali. Da questo punto di vista, l'intervento dello stato dovrebbe rivolgersi maggiormente ai miglioramenti dell'ambiente economico e delle condizioni della domanda.

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THE IMPACT OF MONETARY POLICY IN CFA FRANC ZONE AFRICAN COUNTRIES

by
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I. *Introduction*

The CFA [1] franc zone consists of a number of countries in West and Central Africa whose currencies are firmly tied to the French franc since 1948 [2]. Several studies have analyzed the apparent advantages and disadvantages associated with this type of arrangement in which a number of less developing countries in West and Central Africa have agreed to tie their currencies to that of a single country, France, which is a member of the European Monetary System [3]. The general consensus among researchers is that the advantages of the currency union and monetary standard outweigh the disadvantages. Most recently, Boughton (1991) pointed out that the CFA franc zone countries have gained both monetary discipline and credibility in the formulation of macroeconomic policies.

Monetary discipline and credibility imply that the monetary authorities in these countries follow a systematic and consistent monetary policy actions. According to Boughton (1991), monetary discipline is maintained through (a) interest rate charges on overdraft, (b) rules that require the

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[1] CFA is an abbreviation for « le franc des Colonies Françaises d'Afrique ».

[2] For an overview of the origins and development of the franc zone, see BOUGHTON (1991).

[3] For further readings on the CFA franc zone, see the studies by DEVARAJAN and DE MELO (1987a, 1987b, 1990); and DEVARAJAN and RODRIK (1991).

regional central banks to restrict credit when balances fall below target levels, and (c) the limit on the extension of credit to each government to a maximum of 20 percent of the previous year's fiscal revenue. This argument is also supported by the fact that the currency for these countries is issued by supranational central banks, and France – the anchor country – has relatively strong price stability in the 1980s. The measures of monetary discipline outlined in (a) to (c) above are analogous to the monetary policy rule suggested several years ago by Milton Friedman that a desirable monetary strategy would be to make the money stock grow at a constant rate, which would keep inflation rate close to zero and unemployment close to its natural-rate. According to the monetarists, monetary policy rule elicits trust and eliminates the uncertainty that might confront economic agents about the effects of macroeconomic shocks if monetary policy were to be discretionary. In other words, if expectations are rational, a monetary policy rule anchors economic agents' expectations. Over the years, economists have argued that discretionary monetary policy tends to produce more inflationary pressures with no additional output and employment generated in compensation.

The question one may ask is: Has real output growth responded positively to a systematic money growth rule in these countries? Obviously, one is bound to answer affirmatively on the ground that these countries would have withdrawn their membership in the monetary union if they have to sacrifice output growth. We can also analyze this question by examining the policy implication of the "20 percent rule" on deficit monetization in terms of a non-accommodating or less accomodating monetary policies. Suppose the monetary authorities in the CFA franc zone are committed to a monetary policy of the type $M_t - M_{t-1} = 0$, while the governments adopted tax and expenditure plans that implied large deficits into the indefinite future, that is, $G_t - T_t > 0$ [4]. Monetary and fiscal policies of these nature are incompatible and cannot feasibly be carried out, and may adversely affect the much desired growth. Setting a 20 percent maximum deficit finance on previous year's revenue when monetary policy is $M_t - M_{t-1} > 0$ forces the governments in the zone to exercise fiscal prudence since only 20 percent of government debt can be monetized in any given period. This means that the governments in the zone cannot compel the monetary authorities to monetize larger portions of their deficits as other governments in the neighboring countries do. This is one of the reasons why the CFA franc

[4] This idea is based on the explanation Sargent gave on "Reaganomics and Credibility". For further reading, see SARGENT (1993, pp. 19-41).

zone countries did not experience higher inflation during the 1970s and 1980s in comparison to the neighboring non-CFA franc zone countries where money growth and the production of goods and services have not been kept in balance. As we can see from Table 1, the annual average inflation rates for the CFA countries were substantially lower during the 1970-79 and 1980-89 periods in comparison to the non-CFA countries. While the annual average inflation rates for the CFA countries as a group for these periods were 9.3 percent and 5.8 percent, the annual average inflation rates for the non-CFA countries as a group were 18.4 percent and 40.6 percent for these two periods. Economists have argued that the process of deficits financing through the banking system creates inflationary pressures. Studies have shown that completely monetized budget deficit is one of the major macroeconomic problems in the non-CFA countries.

Using the framework of rational expectations theory, this paper examines the impact of monetary policy on real output growth in the CFA franc

TABLE 1

AVERAGE ANNUAL INFLATION RATES FOR CFA AND NON-CFA COUNTRIES

(a). CFA countries	1970-1979	1980-1989
Benin	8.2	7.7
Burkina Faso	7.9	3.9
Cameroon	10.3	8.5
Côte d'Ivoire	11.7	5.9
Mali	6.2	4.5
Niger	10.4	3.6
Senegal	9.8	6.9
Togo	9.5	5.1
(b). Non-CFA countries		
Ghana	38.8	48.3
Gambia	9.6	17.5
Nigeria	15.7	21.2
Sierra Leone	10.8	63.2
Somalia	10.1	48.6
Tanzania	11.6	30.5
Zaire	40.3	59.5
Zambia	10.3	36.2

SOURCE: Compiled from *International Financial Statistics Yearbook*, 1992.

zone countries listed in the upper part of Table 1. The monetary authorities in the CFA countries follow a systematic money growth rule, therefore, their experience provides a good ground for testing the LSW and NRH-GAP hypotheses. We are well aware that aggregate demand stabilization policy instruments in these countries include fiscal actions, however, our primary focus is on the effects of monetary policy on real output growth because of the uniqueness of these countries. Arguably, the adherence to a systematic money growth rule could be interpreted to mean that these countries would not be able to use rapid monetary expansions to influence real output growth as the other developing countries tend to do.

If the Rational Ecpectations Hypothesis (REH) by Lucas-Sargent-Wallace (LSW) holds in the CFA countries, the systematic (anticipated) changes in money supply would have no effect on real output growth. In contrast, if the Natural Rate Hipothesis-Gradual Adjustment of Prices (NRH-GAP) by Fischer-Phelps-Taylor (FPT) holds, the systematic change in money supply would influence real output growth. In order to find out which hypothesis prevails, we incorporate both hypotheses in this paper as studies have done in the literature to explain real output fluctuations [3]. The rational expectations models developed by LSW posit that anticipated changes in money supply have no effects on real economic variables (see Lucas, 1792, 1973; and Sargent and Wallace, 1975, 1976).

Contrary to LSW's REH, FPT's NRH-GAP postulates that expected changes in money growth are not neutral in the short-run because of price inertia and rigidities in labor contracts (see Fischer, 1977; Phelps and Taylor, 1977; and Gordon, 1982). The test hypothesis in this study is whether the anticipated or/and unanticipated changes in money growth influence real output growth in the CFA franc zone countries. It is possible that monetary discipline and credibility which these countries gained from membership in the zone may nullify the effects of systematic monetary policies such that only monetary surprises can influence real output growth. Our empirical results support the NRH-GAP hypothesis in all the CFA

[3] We do not intend to embark on an extensive literature review that covers the industrialized and less developed countries. For the advanced countries, interested readers should see Barro (1977, 1978, 1979), Barro and Rush (1980), Mussa (1980), Attfield et al. (1981), Gordon (1982), Mishkin (1982a, 1982b), Canarella and Garston (1983), Canarella and Pollard (1989), Merrick (1983), Demery (1984), Darrat (1985), McGee and Stasiak (1985), and Chen and Steindl (1987). For the less developed countries, see Barro (1979), Hanson (1980), Edwards (1983a, 1983b), Attfield and Duck (1983), Kormendi and Meguire (1984), Chopra and Montiel (1986), Montiel (1987), Beladi and Samanta (1988a, 1988b), Al-Saji (1990), and Choudhary and Parai (1991).

franc zone countries except Mali. The results suggest that the countries in the zone do not need monetary surprises to influence real output growth.

The rest of this paper is organized as follows. Section II discusses the data and model specifications. Section III reports the estimated results of the real output growth equation. Section IV is devoted to the policy implications and conclusions of this study.

II. *The Data and Model Specifications*

In order to examine the impact of monetary policy on real output growth in eight CFA franc zone countries in West Africa, this study uses annual time series data. The data for this study include money supply (*M1, M2*), real gross domestic product (*GDP*), exports (*EXP*), imports (*IMP*), merchandise trade and current account balances (*NTB*), government expenditures (*GOV*), real domestic credit (*DCR*), and international reserves (*RES*). The time series data were obtained from various issues of *International Financial Statistics Yearbook* and *Yearbook of National Income Accounts*. We should point out here that the sample period varies across the CFA countries due to data availability.

A. *Time Series Properties of the Variables.* – The important preliminary step to specifying a more detailed model is to examine the data for unit roots. The empirical literature is replete with studies that have shown that the time series process of most economic variables are nonstationary in levels. Granger and Newbold (1974), Stock and Watson (1988), among others, have shown that an ordinary least squares regression of one nonstationary process on several others can generate "spurious correlations" and erroneous conclusions about the regression coefficients. Granger and Newbold (1974) examined some of the consequences of spurious correlations [6]. The most important is that conventional procedures for drawing statistical inferences about estimated coefficients may be invalid in many cases because methods to correct for autocorrelation such as differencing or Cochrane-Orcutt may not be effective.

To avoid the spurious regression problem and the related erroneous inferences, we examined all the variables of interest for unit roots. This was accomplished by using the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) test procedures outlined in Dickey and Fuller (1981). The ADF test is based on a regression of the general form:

[6] See GRANGER and NEWBOLD (1974).

$$\Delta X_t = \pi_0 + \delta X_{t-1} + \sum_{i=1}^{n} \theta_i \Delta X_{t-i} + e_t \qquad (1)$$

where Δ is the first difference operator, $\Delta X_t = X_t - X_{t-1}$. Here, X_t represents the relevant macro variables used in this paper, π_0, δ, θ, are the coefficients, and e_t is the error term. The null hypothesis is that X_t contains a unit root, therefore it is nonstationary [7]. This hypothesis is rejected if δ is significantly negative and the t-value is less than the critical values given by Dickey and Fuller at some desired significance levels. The results of equation (1) are presented in Table 2. When we applied the tests to the log levels of each series, the results suggested that the null hypothesis of a unit root in each variable could not be rejected. This implied that the variables

TABLE 2

RESULTS OF UNIT ROOT TESTS

		M1	*M2*	*GDP*	*GOV*	*DCR*	*EXP*	*IMP*	*NTB*	*RES*
					ADF					
Benin	L.	−2.83 †	−0.18	0.16	1.64	−0.25	−1.43	−0.49	−2.23	−1.92
	D.	−5.40 *	−2.81 †	−3.28 ∓	−2.74 †	−2.69 †	−4.63 *	−6.50 *	−4.97 *	−5.23 *
B-Faso	L.	0.31	0.39	0.59	−0.33	−0.90	−0.63	0.27	−1.72	−0.28
	D.	−3.68 ∓	−4.16 *	−3.80 *	−3.10 ∓	−2.72 †	−4.46 *	−4.00 *	−4.67 *	−4.21 *
Camer.	L.	−1.23	−1.15	−1.13	−0.73	−1.16	−1.38	−1.25	−2.10	−1.68
	D.	−2.86 †	−2.78 †	−3.80 *	−2.71 †	−2.68 †	−3.87 *	−3.24 ∓	−6.69 *	−4.94 *
Côte d'Ivo.	L.	−0.98	−1.07	−0.63	−1.39	−0.97	−0.44	−1.37	−1.92	−1.88
	D.	−2.77 †	−3.15 ∓	−4.91 *	−3.04 ∓	−3.12 ∓	−4.88 *	−2.87 †	−4.04 *	−4.52 *
Mali	L.	−0.01	−0.01	2.24	−0.11	−1.88	0.43	0.42	−2.61	−1.90
	D.	−4.03 *	−4.49 *	−6.91 *	−3.87 *	−4.66 *	−5.66 *	−4.14 *	−5.75 *	−5.51 *
Niger	L.	−0.88	−0.73	−0.49	−1.67	−0.55	−0.58	−0.71	−2.74 †	−0.53
	D.	−3.20 †	−3.32 †	−3.83 *	−3.22 ∓	−3.91 *	−3.92 *	−3.18 ∓	−4.00 *	−3.85 *
Seneg.	L.	0.55	0.46	2.27	0.51	−0.16	−0.01	−0.36	−1.57	−0.09
	D.	−2.67 †	−3.43 ∓	−4.04 *	−2.68 †	−2.86 †	−4.06 *	−3.56 ∓	−3.02 ∓	−4.23 *
Togo	L.	−1.90	−1.76	−1.52	−0.93	−2.43	−1.38	−1.07	−2.54	−1.26
	D.	−5.90 *	−5.44 *	−4.84 *	−3.35 *	−4.86 *	−5.58 *	−3.25 ∓	−5.25 *	−4.95 *

NOTE: L represents levels, and D represents the first difference of the variables. In all cases, regression is performed on the logarithms of the series. Numbers reported are the values of the computed t-statistics for δ. Statistically significant values based on FULLER (1976).

* means significant at the 1 percent level.

∓ means significant at the 5 percent level.

† means significant at the 10 percent level.

[7] A variable X is integrated of order one, I(1), if it is nonstationary and can be written as $X_t = \beta + X_{t-1} + u_t$, where u_t has mean zero and variance σ_u that is stationary. A stationary series is integrated of order zero, I(0). Unit root refers to the coefficient on X_{t-1}.

are nonstationary. However, when the test was applied to the log first difference of each series, the null hypothesis was rejected. This implied that first differencing was required to achieve stationarity. Since the results in Table 2 show that first differences of the series are stationary indicating that all series are integrated of order one, I(1), the money and real output growth equations used in this study were specified in log first differences.

B. *Money Growth Equation.* – Most previous studies have included macro variables in the money growth equation of developing countries that development economists would easily classify as growth-oriented variables [8]. This is because no unique theory exists for predicting money growth in less developing countries. Based on the increasing desire for growth and development, one can safely conclude that the primary objective of the monetary and fiscal authorities in less developed countries is economic growth. Arguably, the CFA franc zone countries in our sample are no exception to the rule. For example, over the past two decades, they have engaged in ambitious growth programs, therefore it is logical to assume that money growth in these countries is linked to the growth rates of real income, the levels of exports and imports, merchandise trade and current account balances, government expenditure, and real domestic credit.

To examine the impact of money growth on real output in these economies, we begin with a money growth equation which can be expressed as:

$$M_t = \Psi_0 + \Psi_1(L)\, M_t + \Psi_2(L)\, \Phi_t + \eta t + e_t \qquad (2)$$

where M_t is the first difference of the natural logarithms of either $M1$ or $M2$ definitions of money supply, Φ_t is a row vector of all relevant macroeconomic variables (in Δ log form) that Granger-cause M_t, $\psi_i(L)$ is the polynomial lag operator associated with M_t and Φ_t, t is the time trend variable, and e_t is the error term.

Following the procedure suggested by Mishkin (1982a, 1982b) with some modifications, we decomposed the money growth into its anticipated and unanticipated components for each country. This involved running a multivariate Granger (1969) tests where the money growth, M_t, was regressed on its own m lagged values and the m lagged values of all relevant

[8] See BELADI and SAMANTA's (1988a) money growth equation for India. CHOUDHARY and PARAI (1991) used MISHKIN's (1982a, 1982b) "atheoretical statistical technique" of running multivariate Granger-causality tests to determine the money growth equation for 13 Latin American countries.

macroeconomic variables contained in the row vector, Φ_t. The m lagged values of M_t and the variables in Φ_t are retained in the money growth for each country if they are jointly significant at the 5 percent level and better.

Several studies have shown that the statistical inferences from causality tests are responsive to lag selection. There are various methods (e.g. Akaike Information Criterion) to determine the optimal lag structures, however, no particular method is superior. To determine the optimal lag length of each variable included in the money growth equation, we followed the multivariate Granger-causality tests with Theil's adjusted R^2 procedure [9]. The procedure works as follows: the lag length for M_t was determined by varying the lag in the autoregressive specification:

$$M_t = \Psi_0 + \Psi_1(L) M_t + e_t \tag{3}$$

from 1 to m where $\psi_1(L) = \Sigma\psi_{1j}(L^m)$, and m is chosen as the maximum lag length. With the lag length of M_t determined, other relevant macro variables were added one at a time. The first macro variable added led to the estimation of a bivariate equation of the form:

$$M_t = \Psi_0 + \Psi_1(L) M_t + \Psi_2(L) \Phi_t + e_t \tag{4}$$

where $\psi_1(L)$ is fixed at its previously determined length from the autoregressive specification given in equation (3). The lags in $\psi_2(L)$ are varied from 1 to m. The maximum lag length is chosen using the highest adjusted R^2 and the F-test criteria.

The macro variable in $\psi_2(L)$ is retained for consideration with other variables if the coefficients are jointly significant at the 5 percent level or better. With the lags in $\psi_1(L)$ and $\psi_2(L)$ fixed at their previously determined lengths in the bivariate equation, the determination of the lag lengths of other relevant variables follows the same procedure outlined above until the appropriate money growth equation is specified for each country [10]. The results for equation (4) are presented in Table 3. The R^2 showed that the variations in money growth are well explained by the variables of each equation. Instead of reporting the Durbin-Watson statistic,

[9] LAUMAS and MCMILLIN (1984), LAUMAS (1991), and CHOUDHARY and PARAI (1991) used the same procedure for their money growth equation.

[10] MISHKIN (1982a, 1982b) argued that this technique imposes a discipline on the researcher that prevents his searching for a money growth specification that leads to results that confirm his priors on the validity of the Macro Rational Expectations (MRE) hypothesis. In addition, he pointed out that the addition of irrelevant variables to an estimated model would lead to a potential decrease in the power of the tests and that the exclusion of relevant variables would render test statistics invalid.

TABLE 3

MONEY GROWTH EQUATIONS

Country (Year)	Equations	R^2	F-stat	Q-stat
Côte d'Ivoire (64-88)	$M_t = -0.42 - 1.52M_{t-1} + 2.66M_{t-2} + 1.16M_{t-3}$ (1.78) (3.25) * (3.30) * (2.99) * $- 0.01GOV_{t-1} + 0.003GOV_{t-2} - 0.01GOV_{t-3}$ (2.08) ∔ (1.32) (2.02) ∔ $- 0.001GOV_{t-4} + 0.009DCR_{t-1} + 0.002DCR_{t-3}$ (0.33) (1.68) † (1.03) $- 0.001RES_{t-1} - 0.002RES_{t-2} - 0.0007RES_{t-3}$ (2.22) ∔ (2.92) * (0.97) $+ 0.0005RES_{t-4} + 1.63NTB_{t-1} - 0.74NTB_{t-2}$ (0.68) (3.73) * (0.67) $- 2.03NTB_{t-4}$ (4.65) *	0.99	84.1	4.2
Senegal (64-86)	$M_t = 1.74 - 0.56M_{t-1} - 0.66M_{t-2} + 0.64M_{t-4}$ (49.5) (20.37) * (28.60) * (18.72) * $-0.002GOV_{t-2} - 0.02GOV_{t-3} - 0.03GOV_{t-4}$ (3.33) * (36.94) * (22.98) * $+ 0.009DCR_{t-1} + 0.001DCR_{t-3} + 0.004RES_{t-2}$ (42.74) * (5.32) * (15.54) * $+ 0.003RES_{t-3} + 0.490NTB_{t-1} + 0.367NTB_{t-2}$ (9.71) * (30.72) * (17.42) * $- 0.223NTB_{t-3} + 0.199NTB_{t-4}$ (17.51) * (14.40) *	0.99	2657.6	3.5
Niger (64-89)	$M_t = -1.51 - 0.284M_{t-1} + 0.723M_{t-3} + 0.11GOV_{t-1}$ (1.92) (1.28) * (2.85) * (0.48) $-0.01GOV_{t-2} - 0.12GOV_{t-3} + 0.016GOV_{t-4}$ (1.72) † (1.70) † (3.16) * $+ 0.307DCR_{t-1} - 0.007DCR_{t-2} + 0.001DCR_{t-3}$ (4.98) * (3.01) * (0.64) $+ 0.005DCR_{t-4} + 0.119RES_{t-1} + 0.0014RES_{t-3}$ (1.54) (4.53) * (1.46) $+ 0.175NTB_{t-4}$ (2.13) ∔	0.99	226.5	5.1
Burkina Faso (66-89)	$M_t = -0.05 + 0.41M_{t-1} + 0.27M_{t-2} - 0.0011GOV_{t-1}$ (0.28) (1.61) † (1.13) (0.20) $-0.001RES_{t-3} + 0.002RES_{t-4} - 0.147NTB_{t-1}$ (0.37) (1.01) (1.61) † $- 0.243NTB_{t-2} - 0.139NTB_{t-3} - 0.471NTB_{t-4}$ (2.19) ∔ (1.10) (2.78) *	0.99	369.7	4.8
Mali (66-90)	$M_t = 0.19 + 1.07M_{t-1} - 0.003RES_{t-1} - 0.001RES_{t-4}$ (2.86) (7.45) * (2.31) ∔ (0.93) *	0.79	28.2	5.1
Togo (64-89)	$M_t = -0.36 + 0.79M_{t-3} + 0.002GOV_{t-1} - 0.013GOV_{t-3}$ (2.29) (0.96) (1.83) † (5.18) * $+ 0.0089DCR_{t-2} + 0.0001DCR_{t-3} + 0.0085DCR_{t-4}$ (1.53) (0.02) (1.42) $+ 0.3609NTB_{t-2}$ (0.74)	0.91	25.6	6.2
Benin (66-87)	$M_t = -0.12 + 0.345M_{t-1} + 0.178M_{t-2} + 0.244M_{t-4}$ (0.32) (1.07) (0.49) (0.80) $-0.003DCR_{t-2} + 0.004RES_{t-1} + 0.001RES_{t-4}$ (0.73) (0.85) (0.28) $+ 0.007EXP_{t-4} + 0.001IMP_{t-2} + 0.002IMP_{t-3}$ (0.33) (0.17) (0.56)	0.83	10.3	3.3
Cameroon (65-90)	$M_t = 0.31 + 0.106M_{t-1} + 0.876M_{t-2} + 0.001GOV_{t-1}$ (7.20) (0.77) (6.64) * (1.87) † $-0.002GOV_{t-2} - 0.0002GOV_{t-3} - 0.0023GOV_{t-4}$ (4.98) * (0.42) (3.45) * $+ 0.0003DCR_{t-2} + 0.0012DCR_{t-3} + 0.0015DCR_{t-4}$ (1.16) (2.19) ∔ (4.04) * $- 0.0003RES_{t-1} - 0.0017RES_{t-2} - 0.0003RES_{t-3}$ (0.97) (4.98) * (1.04) $- 0.0013RES_{t-4} - 0.324NTB_{t-2}$ (4.07) * (5.19) *	0.99	240.2	3.8

NOTES: M represents monetary aggregates, DCR is real domestic credit, GOV is government spending, EXP is exports, IMP is imports, NTB is net trade balance on merchandise and current accounts, RES is international reserves. The number in parentheses are t-statistics with levels of significance: * means significant at the 1 percent level. ∔ means significant at the 5 percent level. † means significant at the 10 percent level.

we report the F-test and the Box-Pierce Q statistics to check the specification of the money growth equation because the Durbin-Watson statistic is known to be biased toward accepting the null hypothesis of no autocorrelation in the residuals when there are lagged dependent variables in the equation (see Durbin, 1970). These test statistics show no evidence of the problem of serial correlation in the money growth equation for any country in our sample.

It is important to point out that our money growth equation for each country in the zone is insensitive to *M1* or *M2* definitions of money. This can be attribued to the fact that the financial markets in the CFA franc zone countries are not adequately developed [11]. In addition, we should also point out that we used the monetary base (domestic credit + foreign reserves) as a measure of money supply as suggested in several studies in the literature. The results with monetary base did not outperform the *M1* or *M2* definition of money supply. Overall, the results in Table 3 suggest that real domestic credit, government expenditure, net trade balance on current merchandise and capital accounts, and international reserves are the key determinants of money growth in these countries.

While the results of the ADF tests in Table 2 show that the variables are nonstationary in logarithm levels but stationary in logarithm first differences, the results in Table 3 show the dynamics of money growth in each country. Although the logarithm first differences are stationary, we can consider that if the variables are not cointegrated, any linear combination of them will be nonstationary, and thus the residuals will be nonstationary. Based on this, we test whether the variables of the money growth equations are indeed cointegrated [12]. Following the theory set forth in Engle and Granger (1987), the general form of the cointegrating regression is specified as:

$$M_t = \varphi_0 + \gamma \Omega_t + u_t \tag{5}$$

where M_t is as defined in previous equations, Ω_t is a row vector of the explanatory variables of the money growth equation as reported in Table 3 for each country in the sample, and u_t is the residual term to be tested. Basically, we test the hypothesis that u_t is not stationary. This is the hypothesis of no cointegration.

[11] Empirical studies on the financial markets of developing countries have not provided any clear-cut evidence on which monetary aggregate economic agents regard as relatively more important for gauging changes in monetary policy.

[12] We are grateful to an anonymous referee for suggesting this additional direction of investigation.

TABLE 4

RESULTS OF THE COINTEGRATING REGRESSIONS

Countries	Dickey-Fuller *t*-statistics
Benin	−5.0389 ≠
Burkina Faso	−4.7494 †
Cameroon	−5.6599 ≠
Côte d'Ivoire	−4.9000 †
Mali	−4.8365 †
Niger	−5.5773 ≠
Senegal	−5.2276 ≠
Togo	−4.2126 †

NOTE:
* means significant at the 1 percent level.
≠ means significant at the 5 percent level.
† means significant at the 10 percent level.

The results of the cointegrating regressions are presented in Table 4. Based on these results, we reject the hypothesis of no cointegration between the variables of the money growth equations. The results of the unit root tests in Table 2 and the cointegrating regressions in Table 4 show ample evidence of the absence of spurious correlations, therefore, the money growth equation for each country is not only robust for our analysis of real output growth equation, but also robust for policy analysis.

C. *Real Output Growth Equation.* – To complete our analysis of the impact of monetary policy on real output growth for the CFA franc zone countries, we specify the real output growth equation in which the explanatory variables are the current and lagged values of the anticipated and unanticipated components of money growth, the lagged values of real output growth, and the time trend. The lagged real output growth captures the dynamics of the economy, while the trend term measures the effects of natural output growth rate. The real output equation takes the general form:

$$Y_t = \lambda_0 + \sum_{i=0}^{N} \alpha_i AM_{t-i} + \sum_{1=0}^{N} \beta_i UM_{t-1} + \sum_{i=1}^{K} \gamma_i Y_{t-i} + \varphi t + \varepsilon_t \qquad (6)$$

where Y_t is the difference in the log of real gross domestic product, AM is the anticipated component of monetary policy (the predicted values from the money growth equation), UM is the unanticipated part of money growth

TABLE 5

REAL OUTPUT GROWTH EQUATIONS

	Country (Year)			
	Benin (1966-87)	Burkina Faso (1966-89)	Cameroon (1965-90)	Côte d'Ivoire (1964-88)
λ	1.663 (7.494) **	7.039 (2.379) **	0.020 (0.268)	−0.178 (1.827) †
AM_t	0.771 (8.580) **	−12.231 (2.484) **	0.495 (3.616) **	0.491 (6.226) **
AM_{t-1}	0.833 (8.457) **	−5.756 (2.091) **	0.166 (0.757)	0.130 (0.765)
AM_{t-2}	0.708 (8.166) **	27.697 (5.538) **	−0.343 (1.657) †	−0.257 (1.868) †
AM_{t-3}	−0.096 (1.539)	−15.109 (5.401) **	−0.273 (1.131)	−0.204 (1.247)
AM_{t-4}	−0.737 (5.4306) **	1.499 (0.596)	0.015 (0.093)	0.296 (2.295) **
UM_t	7.448 (6.589) **	−34.233 (4.256) **	0.240 (0.354)	−0.266 (0.146)
UM_{t-1}	5.691 (10.430) **	−41.065 (3.847) **	0.623 (0.836)	−0.304 (0.143)
UM_{t-2}	8.018 (9.492) **	−49.337 (3.162) **	0.919 (1.334)	2.974 (1.376)
UM_{t-3}	2.310 (6.561) **	−47.145 (2.752) **	1.145 (1.797) *	1.985 (1.007)
UM_{t-4}	2.705 (7.495) **	−32.602 (2.550) **	1.099 (2.005) **	−2.754 (2.052) **
Y_{t-1}	−1.841 (7.685) **	—	0.024 (0.069)	−0.294 (1.088)
t	0.0001 (6.474) **	−0.003 (2.460) **	— —	−0.0001 (4.341) **
R^2	0.95	0.78	0.81	0.93
D.W.	2.5	2.4	1.8	1.7
F-statistics	8.4 **	7.2 **	6.9 **	5.5 **

t-values in parentheses.
** indicates significant at 5% level and better.
† indicates significant at 10% level.

TABLE 5 (*continued*)

	Country (Year)			
	Mali (1966-90)	Niger (1964-89)	Senegal (1964-86)	Togo (1965-89)
λ	−0.077 (1.372)	1.553 (3.321) **	0.305 (4.444) **	0.102 (2.028) **
AM_t	0.241 (1.072)	0.587 (3.466) **	−0.303 (3.205) **	0.079 (1.067)
AM_{t-1}	0.018 (0.053)	−0.169 (0.702)	0.314 (3.732) **	−0.151 (1.782) †
AM_{t-2}	−0.126 (0.315)	0.390 (1.736) †	0.219 (1.911) †	0.059 (0.953)
AM_{t-3}	−0.222 (0.518)	−0.089 (0.353)	− 0.198 (2.461) **	−0.904 (13.460) **
AM_{t-4}	0.164 (0.674)	0.030 (0.176)	−0.166 (1.832) †	0.878 (13.553) **
UM_t	0.372 (0.306)	−1.127 (1.429)	−6.594 (5.616)	−0.003 (0.001)
UM_{t-1}	0.155 (0.805)	1.624 (2.476) **	−10.049 (6.004) **	1.939 (0.477)
UM_{t-2}	−0.061 (0.292)	0.299 (0.358)	−2.895 (1.931) †	0.973 (0.224)
UM_{t-3}	0.266 (0.998)	0.442 (0.611)	−1.449 (1.200)	−1.717 (0.563)
UM_{t-4}	0.235 (0.866)	1.417 (2.032) **	3.134 (2.805) **	−3.494 (0.993)
Y_{t-1}	—	−0.320 (1.019)	−0.596 (4.095) **	—
t	0.00002 (2.107) **	−0.0002 (3.355) **	—	—
R^2	0.59	0.76	0.85	0.97
D.W.	2.1	2.4	2.5	1.5
F-statistics	0.9	7.4 **	6.1 **	5.8 **

t-values in parentheses.
** indicates significant at 5% level and better.
† indicates significant at 10% level.

(the residual from the money growth equation), t is the time trend, and ε_t is the error term.

Before we discuss the results of the real output growth equation, we can simply analyze the hypotheses to be tested in terms of the restrictions placed on the coefficients of AM_t and UMt in equation (6). According to the LSW hypothesis, $\alpha_i = 0$ for all i, $\beta_i \neq 0$ for some of i, and $\Sigma\beta_i = 0$. With respect to the NRH-GAP hypothesis, $\alpha_i \neq 0$ and $\beta_i \neq 0$ for some of i, $\Sigma\alpha_i = \Sigma\beta_i = 0$, and $\alpha_i = \beta_i$ for all of i. Basically, our tests focus on the significance and insignificance of these coefficients, especially $\alpha_i = 0$. For example, if $\alpha_i = 0$ for all of i, we would conclude that the systematic monetary policy actions do not influence real output growth in the CFA countries. On the other hand, if $\alpha_i \neq 0$ and $\beta_i \neq 0$ for some of i, then we can safely conclude that the anticipated and unanticipated components of monetary policies both influence real output growth in CFA countries.

Following the procedure we used for the money growth equation, we estimated variants of equation (6) for each country in our sample. We chose the lag length for AM_t and UM_t based on the highest adjusted R^2. Additional lags of AM_t and UM_t showed no perceived effects on real output growth for lags greater than 4 for all countries.

III. *Empirical Results*

The real output growth equation was estimated by ordinary least squares (OLS) method. The results are presented in Table 5. The adjusted R^2 showed that more than 59% of the variation in real output growth is explained for each country. The Durbin-Watson statistic indicates the absence of serial correlation in the real output growth equations. In addition, the F-statistics – the results of the restrictions implied by LSW hypothesis – for the marginal significance of the set of α_i show that the effect of anticipated money growth on real output is significantly different from zero for seven of the eight countries in our sample. The empirical results reveal similar patterns of effects for the CFA franc zone countries. Specifically, in six of the eight CFA franc zone countries, both the anticipated and unanticipated monetary policy variables have impact on real output growth. In the case of Togo, only the anticipated component of money growth influence real output growth. The parameter estimates for Mali show the time trend variable to be the only significant variable. This result suggests that output grows at its natural rate. The similarity of the effects of money growth on real output for seven of the eight CFA franc zone countries in

our sample is consistent with the views expressed by Boughton (1991) and others that these countries respond to exogenous aggregate demand shocks the same way [13].

In order to draw any valid policy conclusions from these results, one cannot ignore the issue concerning their reliability because of the questions raised by Pagan (1984) and McAleer and McKenzie (1991) with respect to the significance of t-ratios, the consistency of parameter estimates, and the validity of inferences drawn with the standard errors or t-ratios when the regressors of an equation – such as equation (6) – are generated from the predicted values and the residuals of another equation – such as equation (4). Rather than concentrate on the t-ratios of AM_{t-i} as the only basis of

TABLE 6

RESULTS OF TESTING THE RESTRICTIONS IMPLIED BY LSW AND NRH-GAP HYPOTHESES

	Tests			
	Joint Hypothesis		Neutrality	Homogeneity
	$\alpha_i = 0$	$\beta_i = 0$	$\Sigma\alpha_i = \Sigma\beta_i = 0$	$\alpha_i = \beta_i \; \forall_i$
Country	χ^2	χ^2	χ^2	χ^2
Benin	16.02 **	28.89 **	0.98	1.67
Burkina Faso	8.33 **	12.22 **	0.98	0.87
Cameroon	9.23 **	7.88 **	0.81	0.65
Côte d'Ivoire	8.84 **	10.48 **	0.89	0.77
Mali	NA	NA	NA	NA
Niger	9.06 **	12.45 **	1.78	1.08
Senegal	19.86 **	15.67 **	1.69	1.78
Togo	31.44 **	30.69 **	1.09	1.49

NOTES: NA denotes that the statistic cannot be computed because the real output growth result is inconclusive.

** indicates rejection of the null hypothesis at the 5% level.

* indicates rejection of the null hypothesis at the 10% level.

[13] See BOUGHTON (1991).

our rejection of LSW hypothesis or the acceptance of NRH-GAP hypothesis, we performed the following tests: (a) Joint significance hypothesis: $\alpha_i = 0$, and $\beta_i = 0$, (b) Neutrality hypothesis: $\Sigma\alpha_i = 0$, and $\Sigma\beta_i = 0$, and (c) Homogeneity hypothesis: $\alpha_i = \beta_i$ for all of i.

The joint significance hypothesis test is conducted to examine whether AM_{t-i} and UM_{t-i} have significant effects on real output growth in each country. The long-run neutrality test is a test of whether the long-run effect of money on real output growth is zero. The homogeneity test is the test as to whether expectations matter. Recent empirical evidence on the latter test by Frydman and Rappoport (1987) showed that the decomposition of money growth into anticipated and unanticipated components is statistically irrelevant in explaining the movements in real output growth. The results of these tests are presented in Table 6. We reject the null hypotheses in columns 2 and 3 for all the CFA countries in our sample except for Mali whose inconclusive output growth result prevented further tests. In columns 4 and 5, we failed to reject the long-run money neutrality and homogeneity hypotheses. In other words, the long-run neutrality of money holds in these

TABLE 7

DIAGNOSTIC TESTS OF THE REAL OUTPUT GROWTH EQUATION

Country	Diagnostic Tests			
	Serial Correlation	White's Hetero-skedasticity	RESET	Stability
Benin	0.76	1.85	0.85	0.65
Burkina Faso	1.45	0.78	0.98	1.45
Cameroon	1.79	1.72	1.02	1.09
Côte d'Ivoire	1.65	0.60	1.94	1.09
Mali	NA	NA	NA	NA
Niger	1.09	1.54	1.55	0.65
Senegal	1.23	1.86	0.92	0.98
Togo	1.53	0.69	1.40	2.09

NOTES: NA denotes that the statistic cannot be computed because the real output growth result is inconclusive.

** indicates statistically significant at the 5% level.

* indicates statistically significant at the 10% level.

countries, and the decomposition of money growth into anticipated and unanticipated components is not necessary to explain movements in real output growth.

It is not uncommon in applied econometric research to estimate spurious regression and obtain the correct signs with high R^2. Granger and Newbold (1974) have addressed this type of problem. Several studies have suggested that conventional regression output should be supplemented with a battery of diagnostic tests if one is to draw valid inferences. To solve this problem, we performed the Breusch-Godfrey's serial correlation (LM), White's heteroskedasticity, Ramsey's RESET, and Chow's stability tests. These results are provided in Table 7. The results of these battery of diagnostic tests for the real output equation show that there are no serial correlation or heteroskedasticity problems, no functional misspecification, and no structural instability in the regressions. These tests further lend credence to our empirical findings.

IV. *Policy Implications and Conclusions*

This paper has investigated the impact of monetary policy on real output growth for eight CFA franc zone countries in West Africa where the monetary authorities in these countries have monetary discipline and credibility. The purpose was to test whether systematic money growth influence real output growth in these countries by employing the REH and NRH-GAP models. This was accomplished by decomposing money growth into its anticipated and unanticipated components for each country in the sample. Our empirical results indicate similar patterns of effects for the CFA franc zone countries. With the exception of Mali whose results are inconclusive, our empirical results indicate that under a systematic monetary policy rule regime, the effects of anticipated monetary policies on real output growth are not neutral in the short-run.

In addition to the fact that the empirical results support the NRH-GAP, the results of our long-run money neutrality and homogeneity tests suggest that the long-run neutrality of money holds and that anticipated and unanticipated monetary policies, regardless of how they are measured, affect real output growth in the CFA franc zone African countries in the sample. In other words, under a disciplined and credible monetary policy regime, the anticipated and unanticipated monetary policies will influence real output growth, therefore, monetary authorities do not need monetary surprises for economic stability. Finally, if we compare the inflation rates in the CFA

countries (see Table 1) with those of the non-CFA countries, this study shows that monetary policy can be an effective aggregate demand stimulus with little or no inflationary pressures under a systematic monetary policy rule.

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L'IMPATTO DELLA POLITICA MONETARIA NEI PAESI AFRICANI DELLA ZONA DEL FRANCO

Questo articolo esamina l'impatto della politica monetaria sulla crescita reale della produzione in otto paesi dell'Africa occidentale (zona del franco) dove viene mantenuta una disciplina monetaria. A questo scopo viene applicata l'ipotesi delle aspettative razionali (REH) e l'ipotesi del tasso naturale con un aggiustamento graduale di breve periodo dei prezzi alle fluttuazioni reali della produzione in questi paesi (ipotesi NRH-GAP).

Queste sono le due ipotesi largamente usate nella letteratura per esaminare le fluttuazioni reali della produzione in molti paesi sviluppati e in via di sviluppo. L'ipotesi RAH (avanzata da Lucas-Sargent-Wallace) presuppone che i cambiamenti previsti della crescita monetaria non abbiano alcun effetto sulle variabili economiche reali. Invece la NRH-GAP (avanzata da Fischer-Phelps-Taylor) postula che una crescita monetaria prevista non sia neutrale nel breve periodo a causa delle inerzie dei prezzi e delle rigidità nei contratti di lavoro. I risultati empirici per i paesi africani della zona del franco francese mostrano che le componenti della crescita monetaria sia prevista sia non prevista influiscano sulla crescita reale della produzione. I nostri risultati mostrano anche che in questi paesi vale la neutralità di lungo periodo della moneta. Inoltre, i risultati dei test di omogeneità suggeriscono che la scomposizione della crescita monetaria nelle sue componenti previste e non previste non è necessaria per spiegare la crescita reale della produzione in questi paesi. I nostri risultati empirici avvalorano l'ipotesi NRH-GAP.

RECENSIONI

NARDI SPILLER Cristina: *La dinamica inflativa nell'economia italiana dal secondo dopoguerra agli anni Novanta.* Padova, Cedam, 1994, pp. X-167, L. 22.000.

Il volume muove dall'esigenza, avvertita da C. Nardi Spiller, di indagare l'evoluzione dell'economia italiana, proiettando lo studio soprattutto in merito alla sperimentata e variegata dinamica inflativa, per poterne offrire un quadro di analisi interpretativa. D'altronde, convinzione dell'Autrice, è che i fenomeni, pur nascendo isolatamente, si possono tra loro intrecciare, creando interdipendenze spesso difficilmente individuabili sotto il profilo teorico e altrettanto insidiose sotto quello operativo.

Nel 1945, l'Italia riveste la piena peculiarità di un Paese contadino, mentre alle soglie del Duemila sta ormai superando l'« etichettatura » di società industriale per uniformarsi al modello centrato sul terziario. La trasformazione, intervenuta nel corso del tempo, comporta modifiche sostanziali nell'ambito delle imprese, sul versante dei lavoratori, sull'assetto statale e sull'intero sistema economico, dal momento che i cambiamenti intercorsi innescano profonde modificazioni, anche nei rapporti internazionali. In effetti, Nardi Spiller scrive che « le componenti ereditate dal passato, non vanificandosi, divengono parti residuali, ma non marginali, di consistente spessore e dotate di una loro specifica individualità, tali da esercitare una influenza rilevante sull'intero processo evolutivo » (p. 2).

Dal processo inflazionistico, che non è « un gioco a somma zero », in quanto le varie cause ed effetti, interagendo tra loro, producono un risultato ben diverso dalla « semplice » sommatoria delle singole componenti, discendono nodi cruciali che investono, tra l'altro, la distribuzione del reddito e comportano una deformazione, sovente sostanziale, nella struttura dei prezzi relativi. In particolare, con riguardo alla dinamica inflativa, diviene cruciale il rilievo delle aspettative che favoriscono ripercussioni autoalimentantesi sulla stessa spirale inflazionistica. Da un simile complesso scenario, emerge la difficoltà di ricondurre l'inflazione a un solo movente, atto a mettere in moto un unico meccanismo, tanto più che lo svolgimento dei fatti economici « sfugge ad una spiegazione monistica » (Savona P., *Considerazioni sullo stato della modellistica economica in Italia: note a margine dei due modelli CSC*, « Economia Italiana », n. 2). Pertanto l'inflazione, una volta innescata, presenta insidie notevoli, così che appare arduo riportarla sotto controllo.

Il ripercorrere il cammino dell'economia italiana, qui articolato in tre parti distinte a seconda del decennio considerato, conduce l'Autrice ad analizzare parallelamente l'evolversi dei più significativi approcci economici per stimarne la valenza in sede operativa.

Nel dopoguerra, si assiste alla ricostruzione dell'economia italiana, profondamente segnata dal secondo conflitto mondiale: inizia, dall'esame di simili tematiche, la Parte Prima del volume (*L'economia italiana nell'immediato dopoguerra*). Le problematiche, venutesi allora a creare, investono la stessa riconversione produttiva, dal momento che dall'economia di guerra si passa a una economia di pace. Il controllo sul mercato dei beni e del cambio divengono i due temi

centrali di discussione. Tuttavia, per quanto concerne il mercato dei beni, l'orientamento dell'autorità centrale è teso a evitare qualsiasi controllo. Si vuole, in tal senso, distaccarsi completamente dal regime passato che, attraverso il sistema delle corporazioni, instaura una economia largamente controllata. Di qui il vivo desiderio di ritornare a una economia di tipo concorrenziale, sebbene si tema che la politica liberista possa aprire le vie all'inflazione. D'altronde il cambio della moneta, che solleva una vivace *querelle* anche con riguardo alle posizioni politiche, non può essere perseguito per le affermazioni della Banca d'Italia, relative alla mancanza di sicurezza del trasporto della nuova valuta sull'intero territorio nazionale. In secondo luogo, l'avviata integrazione nell'economia mondiale e lo sviluppo veloce delle esportazioni domestiche determinano, seguendo le considerazioni della studiosa, le peculiarità del nostro processo di crescita, il quale trova negli anni che vanno dal 1951 al 1961 (più precisamente dal 1958 al 1961) la realizzazione di un periodo singolarmente felice per il nostro Paese, tanto da essere contraddistinto dalla terminologia « miracolo economico » o « anni del *boom* economico ».

In tale decennio, la cui analisi « apre » la Parte Seconda (*Il grande balzo e la susseguente inversione di rotta*), il mercato del lavoro è inizialmente caratterizzato da una eccesso di offerta di lavoro che segna altresì la debolezza sindacale. Inoltre, all'epoca, alle banche italiane non si presenta una pluralità di scelta per gli impieghi: o trovano modo di collocare la propria liquidità in impieghi presso il settore privato o collocano l'esubero in Buoni del Tesoro. In questo contesto, si registra un notevole sviluppo delle banche minori che, in un certo senso, sottraggono parte del settore bancario alla sfera pubblica dell'economia. In effetti, le grandi banche, in un modo o nell'altro, fanno capo al settore pubblico, mentre le piccole banche locali afferiscono a gruppi finanziatori privati. Come evidenzia Nardi Spiller, nel 1962, si registra un radicale rovesciamento di tendenza nei rapporti industriali. In tale anno, attraverso una contesa sindacale fattasi più aspra, il livello dei salari contrattuali cresce velocemente, con il risultato di estendere gli aumenti salariali, dapprima circoscritti alle imprese maggiori, a una cerchia più vasta di medie e piccole imprese. Di qui dapprima l'instaurarsi di un meccanismo di *demand pull*, provocato dal fenomeno dello slittamento salariale, e poi il sorgere di una inflazione più propriamente da salari, a cui si aggiunge, almeno in una prima fase, anche quella da profitti. L'economia italiana, dopo un decennio di dinamica inflativa impercettibile, passa dunque a una situazione di inflazione vera e propria. Ne derivano inevitabili ripercussioni sulla bilancia commerciale, dal momento che i più elevati prezzi interni vanno a comprimere la nostra competitività. Ma lo scenario economico è sensibilmente influenzato anche da salienti ripercussioni politiche. Infatti, nel 1962, si realizza un governo di coalizione con l'appoggio esterno del partito socialista, mentre nel 1963 tale partito entra al Governo. Ne segue l'adozione di straordinarie misure di intervento, quali la nazionalizzazione delle imprese elettriche. I timori prodottisi sulla conduzione della politica economica nazionale comportano consistenti e destabilizzanti fughe di capitali, espressione della crisi del sistema e causa di ulteriori gravi perturbazioni, come già Fanno illustra nel 1935 (*I trasferimenti anormali dei capitali e le crisi*, Torino, ed. Einaudi). Dopo un susseguirsi di periodi alterni, nel 1965 si avvia una certa ripresa. Gli anni fra il 1965 e il 1968 sono contraddistinti da una sostanziale stabilità monetaria che viene bruscamente interrotta nel 1969. In quest'ultimo anno, si incorre, a parte qualche iniziale fenomeno di portata internazionale dovuto all'impennata di alcuni prezzi agricoli, nell'« autunno caldo », che segna radicalmente la nostra economia, non solo per le accese rivendicazioni salariali, ma anche per il fitto intrecciarsi di varie richieste che si viene a produrre e che modifica sensibilmente i tradizionali rapporti di lavoro. Gli anni Settanta sono poi a loro volta segnati da due eventi straordinari: gli shock petroliferi del 1973 e del 1979, il cui impatto sulle economie occidentali comporta una modifica saliente nel tessuto economico-sociale di ogni

Paese, sollecitando altresì, in sede teorica, una revisione degli schemi analitici precedentemente impiegati. Come ricorda C. Nardi Spiller (che peraltro, sotto il profilo teorico, ben delinea nel suo precedente volume *Struttura produttiva e dinamica dei prezzi*, Padova, Cedam, 1990), la realtà così maturata può sfociare in una situazione di *stagflation* che diviene le plausibile premessa a stati di *unempflation* e di *slumpflation*, che concorrono a minare le basi del credo keynesiano.

Nella Parte Terza (*Vecchi e nuovi problemi*) si evidenzia come nell'arco temporale considerato si assista a una revisione delle richieste da parte sindacale. Se ciò influisce positivamente sull'andamento economico, è pur vero che tale periodo è contraddistinto da un crescente debito pubblico, che diviene il « problema dei problemi ». Si vengono così a delineare delle anomalie del regime fiscale-monetario che segnano la nostra economia e che possono essere sintetizzate in quattro punti fondamentali. Il debito pubblico italiano risulta molto breve e dunque comporta un'elevata frequenza di rinnovi. I titoli del debito pubblico crescono, in modo vivace, nei portafogli titoli delle famiglie. Si registrano elevati saggi d'interesse reale, nonostante il prevalere del debito a breve termine, più liquido. Infine, gli effetti « spiazzamento » (operati dal comparto pubblico a danno di quello privato) non sono particolarmente significativi. L'inizio degli anni Novanta è caratterizzato da tre importanti avvenimenti che si impongono sulla scena mondiale. I nuovi assetti politici dell'Est europeo, culminanti con l'unione della Germania, la « crisi del Golfo », definita « tempesta nel deserto », il rallentamento ciclico che, pur atteso da tempo, si mostra più pronunciato del previsto. Dal protrarsi della recessione negli Stati Uniti, dall'arresto dell'espansione in Germania e in Giappone, nonché dalla disgregazione monetario-economica dell'Europa Centro-Orientale provengono tensioni che minano ulteriormente la fase recessiva che investe i Paesi industriali. « Con riguardo all'Italia, unitamente alle problematiche connesse alla recessione internazionale, scrive l'Autrice, si registrano, nel 1991, carenze nella gestione di politica economica, si rileva una eccessiva crescita del costo del lavoro, si constata l'esistenza di insufficienti iniziative da parte delle imprese, si rivela sempre più preoccupante la situazione del bilancio pubblico: squilibri che, minacciando la nostra struttura produttiva, anche nel suo potenziale sviluppo di lungo periodo, rischiano di precludere il percorso tracciato dall'accordo di Maastricht » (p. 102).

Il testo in esame, che abbiamo qui commentato, rappresenta uno studio attento e critico dei fenomeni economici più significativi che interessano il nostro Paese e che, nel tempo, concorrono a modificarne il relativo tessuto economico-sociale, tanto che, sotto questo profilo, il volume può ben inquadrarsi nell'ambito dei contributi sulle fluttuazioni economiche. Lo scrupoloso esame condotto (pur caratterizzato dall'obiettività, sebbene le tematiche affrontate siano spesso insidiose) si impernia sulla centralità della dinamica dei prezzi. In tale contesto, le spiegazioni interpretative offerte ai vari processi inflativi sono « incasellate » nei periodi storici in cui manifestano la propria valenza e virulenza. Occorre aggiungere, al riguardo, che nella sua pubblicistica, C. Nardi Spiller è sempre protesa a individuare il contesto storico in cui si maturano delle realtà, sia relativamente agli aspetti reali che teorici. Sotto quest'ultimo profilo, va segnalato che al termine di ogni decennio considerato viene data una sintetica ma ugualmente incisiva illustrazione degli apporti teorici che segnano maggiormente il panorama speculativo dell'analisi economica. Conclude il volume, da indirizzarsi anche a un pubblico interessato a conoscere i problemi italiani, un'Appendice Statistica degna di attenzione.

GIOVANNI ZALIN

ALLEANZA ASSICURAZIONI

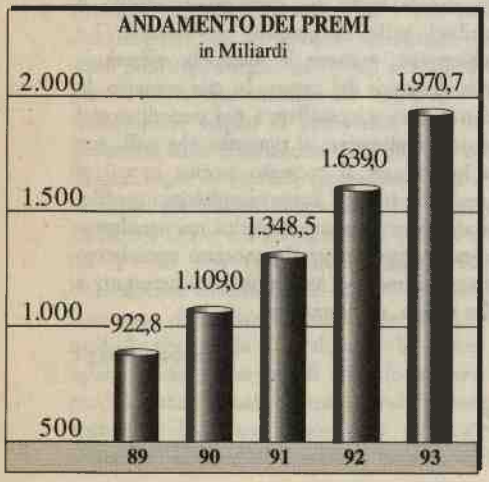
SOCIETÀ PER AZIONI CON SEDE E DIREZIONE GENERALE IN MILANO - VIALE LUIGI STURZO, 35

BILANCIO 1993

ENTRATE	(miliardi di lire)
Premi e accessori	1.970,7
Premi ceduti in riassicurazione	(-),975,5
Redditi ordinari e profitti da alienazione	1.292,9
Saldo prelievi - accantonamenti	(-) 59,8
Altri proventi	43,8
TOTALE	**2.272,1**

USCITE	
Sinistri, scadenze, riscatti	238,4 *
Incrementi riserve tecniche	804,2 *
Redditi verso riassicuratori	341,6
Costi di produzione e amministrazione	187,4 *
Minusvalenze da titoli	107,6
Altri oneri	105,4
Imposte e tasse	299,9
Utile dell'esercizio	**187,6**
TOTALE	**2.272,1**

* *al netto quote cedute ai riassicuratori*



PARTE ORDINARIA

- I premi sono ammontati a 1.970,7 miliardi (1.639 miliardi nel 1992), con un incremento pari al 20,2%. Sono stati ceduti in riassicurazione 975,5 miliardi, pari al 49%.
- La produzione netta in capitali è stata di 8.562 miliardi (7.530), con un aumento del 13,7%, mentre i premi annui di nuova produzione sono ammontati a 485 miliardi (425,6), con un incremento del 13,9%.
- I redditi e profitti da alienazione hanno raggiunto 1.292,9 miliardi (899), con un incremento del 43,8%: sono stati destinati per il 26% al servizio riassicurazione, per il 27% alle riserve tecniche e per il 47% a favore del conto economico. Rendimento Fondo S. Giorgio: 13,88%, retrocessione agli Assicurati 80%.
- Gli investimenti ammontano a 9.680 miliardi (7.979,2 nel 1992), ripartiti come segue: obbligazioni 59,1%, immobili 21,2%, partecipazioni 9,8%, altri 9,9%. Il rendimento medio è del 9,7%.
- I sinistri, le scadenze e i riscatti sono stati di 499,6 miliardi, di cui 238,4 a carico della Società e 261,2 a carico dei riassicuratori.
- A gennaio 1994 si è attuato il progetto di acquisizione della maggioranza assoluta della Venezia S.p.A. e della Fincral S.p.A.. Con il gennaio 1995 l'Alleanza controllerà il 100% del capitale delle suddette Società.
- L'utile è stato di L. 187,6 miliardi. È stato deciso un dividendo di L. 135 sulle azioni ordinarie e di L. 160 sulle azioni di risparmio n.c., pagabili dal 18 luglio; tenuto conto della base azionaria allargata per effetto dell'aumento gratuito del Capitale Sociale realizzato nel 1993, il dividendo è aumentato del 25%. La Riserva Patrimoniale è stata rafforzata di 74 miliardi, raggiungendo 396,4 miliardi.
  L'utile per azione (ordinarie e di risparmio n.c.) è di L. 288,4 e il rapporto dividendo/utile è del 48,3%.
- A integrazione del Consiglio d'Amministrazione sono stati nominati il prof. Giovanni Bazoli, Presidente del Banco Ambrosiano Veneto e il dott. Giuseppe Buoro, Direttore Centrale responsabile delle attività vita delle Assicurazioni Generali.
- Il Bilancio Consolidato ha chiuso l'esercizio 1993 con premi lordi per 1.962,8 miliardi (+ 19,3%), investimenti per 9.666,5 (+ 21,2%) e un saldo utile di 190,2 miliardi.

ALLEANZA
ASSICURAZIONI

Banco di Sardegna S.p.A.

Capogruppo del Gruppo bancario omonimo - N. 1015-7 Reg.
Sede legale Cagliari - Sede amministrativa Sassari
Capitale sociale L. 206.611.220.000
Riserve e altri Fondi patrimoniali L. 1.111.947.757.979
Reg. Soc. Trib. Cagliari 24488 - CCIAA Cagliari 0160586
Aderente al Fondo Interbancario di Tutela dei Depositi

Bilancio dell'esercizio 1993

L'Assemblea generale ordinaria dei soci del Banco di Sardegna S.p.A., riunita in Sassari in seconda convocazione il giorno 9 maggio 1994, ha approvato il bilancio dell'esercizio chiuso al 31 dicembre 1993, assumendo le relative connesse deliberazioni, compresa quella concernente la ripartizione degli utili.

RISULTATI SINTETICI

Raccolta da clientela	9.419	+ 8,1%
Impieghi globali (crediti ordinari e speciali alla clientela)	9.131	+ 9,3%
Risultato lordo di gestione	393	+ 94,9%
Ammortamenti, rettifiche e riprese	172	+ 107,7%
Imposte sul reddito	176	+ 184,0%
Utile netto	45	— 20,8%
Fondi patrimoniali dopo il riparto degli utili	1.319	+ 1,8%

importi in miliardi di lire

Dividendo unitario assegnato:
— L. 600 alle azioni di risparmio;
— L. 500 alle azioni ordinarie.

ISTITUTO CENTRALE DI BANCHE E BANCHIERI

Sede Sociale e Direzione Generale: Milano, Corso Monforte, 34

STATO PATRIMONIALE AL 31 DICEMBRE 1993

ATTIVO

		Lire	Lire
10.	Cassa e disponibilità presso banche centrali e uffici postali		2.051.787.527
20.	Titoli del Tesoro e valori assimilati ammissibili al rifinanziamento presso banche centrali		1.121.655.543.914
30.	Crediti verso enti creditizi:		2.727.551.417.458
	a) a vista	561.380.018.579	
	b) altri crediti	2.166.171.398.879	
40.	Crediti verso clientela		277.018.587.797
50.	Obbligazioni e altri titoli a reddito fisso:		601.407.782.266
	a) di emittenti pubblici	569.726.341.538	
	b) di enti creditizi	31.641.923.678	
	c) di enti finanziari	2.252.250	
	d) di altri emittenti	37.264.800	
60.	Azioni, quote e altri titoli a reddito variabile		87.492.145
70.	Partecipazioni		147.332.866.627
80.	Partecipazioni in imprese del Gruppo		50.554.200.000
90.	Immobilizzazioni immateriali		559.552.427
100.	Immobilizzazioni materiali		122.180.819.416
130.	Altre attività		203.802.768.663
140.	Ratei e risconti attivi:		80.970.707.121
	a) ratei attivi	80.070.239.827	
	b) risconti attivi	900.467.294	
	Totale dell'Attivo		**5.335.173.525.361**

PASSIVO

		Lire	Lire
10.	Debiti verso enti creditizi:		4.505.766.180.908
	a) a vista	965.499.270.371	
	b) a termine o con preavviso	3.540.266.910.537	
20.	Debiti verso clientela		135.411.788.948
	a) a vista	116.243.267.065	
	b) a termine o con preavviso	19.168.521.883	
30.	Debiti rappresentati da titoli:		126.458.892.533
	a) altri titoli	126.458.892.533	
50.	Altre passività		88.976.300.239
60.	Ratei e risconti passivi:		66.862.618.256
	a) ratei passivi	66.787.272.413	
	b) risconti passivi	75.345.843	
70.	Trattamento di fine rapporto di lavoro subordinato		5.644.992.497
80.	Fondi per rischi ed oneri:		24.018.378.981
	b) fondi imposte e tasse	24.018.378.981	
90.	Fondi Rischi su Crediti		1.375.000.000
100.	Fondo per rischi bancari generali		7.109.965.566
120.	Capitale		95.000.000.000
130.	Sovrapprezzi di emissione		6.596.015.000
140.	Riserve:		175.509.255.780
	a) riserva legale	52.063.731.606	
	b) altre riserve	123.445.524.174	
150.	Riserve di rivalutazione		83.378.856.235
170.	Utile d'esercizio		13.065.280.418
	Totale del Passivo		**5.335.173.525.361**

Garanzie e impegni

10.	Garanzie Rilasciate		31.832.668.492
	di cui:		
	– accettazioni	294.344.846	
	– altre garanzie	31.538.323.646	
20.	Impegni		907.843.699.423